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THE FEBRUARY MEETING IN NEW YORK

The two hundred seventy-seventh meeting of the American Mathematical Society was held at Columbia University, on Saturday, February 22, 1930, extending through the usual morning and afternoon sessions. The attendance included the following sixty-nine members:

A. A. Albert, L. B. Andrews, R. G. Archibald, H. E. Arnold, F. W. Beal, Benton, B. H. Camp, Chambers, Cheney, L. W. Cohen, Cooley, C. C. Craig, Demos, Doermann, Donahue, Dresden, Arne Fisher, Fiske, Fite, D. A. Flanders, Flexner, R. M. Foster, C. C. Grove, Hansell, Robert Henderson, L. S. Hill, Himwich, Hofmann, Huber, Huntington, M. I. Johnson, Keffer, Kline, Knebelman, Knobelauch, Koopman, Kramer, Lefschetz, MacColl, S. P. Mead, F. H. Miller, Molina, T. W. Moore, F. H. Murray, C. A. Nelson, Oppenheim, Ore, Parsons, Post, Reddick, Ritt, J. H. Roberts, Robertson, Roos, Ruger, H. M. Schlauch, Seely, Serghiesco, Simons, Smail, P. A. Smith, T. Y. Thomas, Wedderburn, Weida, Weisner, W. A. Wilson, Winters, Margaret M. Young, Zobel.

At the meeting of the Council the following persons were elected to membership in the Society:

Professor Harold Lish Black, Westminster College;
Professor Charles Theodore Bumer, Ohio State University;
Mr. Robert Horton Cameron, Cornell University;
Professor Lewis Townsend Cook, St. Lawrence University;
Mr. John Johnson Corliss, University of Michigan;
Mr. William Larkin Duren, Jr., University of Chicago;
Mr. Edward J. Finan, Ohio State University;
Mr. Dan Hall, Agricultural and Mechanical College of Texas;
Professor Cincinnatus Decatur Killebrew, Alabama Polytechnic Institute;
Professor Herbert Albert Meyer, Hanover College;
Professor Walter McKinley Miller, Marquette University;
Dr. Paul Smith Olmstead, Bell Telephone Laboratories;
Mr. Herbert Dellmont Owens, University of Minnesota;
Professor James V. Uspensky, Stanford University;
Professor William Wilson Weber, Lander College;
Mr. John Henry Constantine Whitehead, Princeton University;
Professor William Miller Wible, Intermountain Union College;
Mr. William Henry Yates, civil engineer, New York City.

Eleven applications for regular membership were received and one nomination of a sustaining member.

The following appointments by the President were an-

nounced: To represent the Society at the semi-centennial celebration of the University of Southern California on June 4, 1930, Professor E. R. Hedrick; to represent the Society on the Joint Committee of Engineering Statistics, Professor E. V. Huntington; as chairman of the subcommittee on mathematics of the National Research Council Science Advisory Committee of the Chicago World's Fair Centennial Celebration in 1933, Professor Huntington; as a committee to consider the establishment of a journal of applied mathematics, Dr. T. C. Fry, chairman, Mr. P. L. Alger, Professor Tomlinson Fort, Mr. Charles Le G. Fortescue, Professors S. Timoshenko, Warren Weaver, and Norbert Wiener.

On the recommendation of the Committee on the Visiting Lecturership, Professor E. Bompiani, of the University of Rome, was appointed as the Society's visiting lecturer for the year 1930.

At a business session of the Society held before the beginning of the regular afternoon session, the following amendments to the By-Laws were adopted, as recommended by the Council: Article VI, Section 2, to read "The annual dues of persons elected by the Council to ordinary membership under the provisions of Article V, Section 4, shall be eight dollars, with the following exceptions: (1) during the first three years of membership, the annual dues shall be six dollars; a person shall be considered to have completed his first year of membership on January 1 following his election; (2) the amount of dues may be altered by reciprocity agreements with other societies; (3) the Council may make special rulings in exceptional cases, with the approval of the Board of Trustees. Each new member shall pay in proportion to the unexpired fraction of the year at the time of his election." Article VI, Section 4, to read "Persons elected to ordinary membership as nominees of sustaining members, under the provisions of Article V, Section 3, shall not be required to pay an initiation fee. They shall not be required to pay dues so long as it is agreed that they are designated by the sustaining member under these provisions. If a nominee of a sustaining member later becomes a dues-paying member, he shall pay dues at the rate of six dollars a year for the remainder (if any) of his first three years of membership, and eight dollars a year thereafter."

At the request of the Program Committee, Mr. E. C. Molina, of the American Telephone and Telegraph Company, delivered, at the afternoon session, an address entitled *The theory of probability; some comments on Laplace's Théorie Analytique.* By the courtesy of the American Telephone and Telegraph Company, this address was printed in advance of its delivery, and copies were distributed at the meeting.

Vice-President Lefschetz presided at the morning session, Mr. Robert Henderson during the business meeting, and Professor Huntington at the regular afternoon session.

Titles and cross-references* to the abstracts of the papers read at this session follow below. The papers whose abstract numbers are followed by the letter *t* were read by title. Mr. Borofsky was introduced by Professor Ritt, and Mr. Smiley by Professor Evans.

1. *Notes on the Jonquière quartic*, by Professor H. E. Arnold. (Abstract No. 36–3–171.)

2. *On the geometry of the second derivative of a polygenic function*, by Professor J. E. Donahue. (Abstract No. 36–3–172.)

3. *Plane transformations preserving centers of gravity*, by Dr. Lulu Hofmann. (Abstract No. 36–3–173.)

4. *A necessary and sufficient condition for the non-equivalence of any two rational generalized quaternion division algebras*, by Dr. A. A. Albert. (Abstract No. 36–3–174.)

5. *On hypergeometric functions of several variables*, by Professor Oystein Ore. (Abstract No. 36–3–175.)

6. *Linear homogeneous differential equations with Dirichlet series as coefficients*, by Mr. Samuel Borofsky. (Abstract No. 36–3–176.)

7. *A point set characterization of 2-dimensional closed manifolds*, by Dr. J. H. Roberts (National Research Fellow). (Abstract No. 36–3–177.)

8. *On the region covered by a certain polynomial*, by Professor J. E. Donahue. (Abstract No. 36–3–178.)

9. *Irreducible non-planar graphs*, by Professors Orrin Frink and P. A. Smith. (Abstract No. 36–3–179.)

10. *A construction of all non-commutative rational division*

* See this Bulletin, vol. 36, p. 6, footnote.

algebras of order eight, by Dr. A. A. Albert. (Abstract No. 36–3–180–*t*.)

11. *The first variation of a functional*, by Professor G. C. Evans and Mr. W. G. Smiley, Jr. (Abstract No. 36–3–181–*t*.)

12. *Geometries in which straight lines are shortest*, by Dr. Lincoln LaPaz. (Abstract No. 36–3–182–*t*.)

13. *A continuum every subcontinuum of which separates the plane*, by Professor G. T. Whyburn. (Abstract No. 36–3–183–*t*.)

14. *Completely partitionable connected sets*, by Professor G. T. Whyburn. (Abstract No. 36–3–184–*t*.)

15. *On non-separated cuttings of connected point sets*, by Professor G. T. Whyburn. (Abstract No. 36–3–185–*t*.)

16. *Concerning certain continuous curves in the plane and connected and connected im kleinen point sets in general*, by Professor G. T. Whyburn. (Abstract No. 36–3–186–*t*.)

17. *A normal form for maps*, by Mr. Hassler Whitney. (Abstract No. 36–3–187–*t*.)

18. *A necessary and sufficient condition for convergence of a sequence of Lagrange polynomials*, by Professor George Rutledge. (Abstract No. 36–3–188–*t*.)

19. *On certain types of surfaces in 3-dimensional euclidean space*, by Dr. T. C. Benton. (Abstract No. 36–3–189–*t*.)

20. *Involutorial transformations of order N with an* (N–1)*-fold line*, by Mr. L. A. Dye. (Abstract No. 36–3–190–*t*.)

21. *Construction of division algebras*, by Professor L. E. Dickson. (Abstract No. 36–3–191–*t*.)

22. *On the overconvergence of sequences of polynomials of best approximation*, by Professor J. L. Walsh. (Abstract No. 36–3–192–*t*.)

23. *The third characteristic property of the derivative of a polygenic function*, by Professor Edward Kasner. (Abstract No. 36–3–193–*t*.)

24. *Rotational symmetry in the characteristic functions of the Schrödinger wave equation*, by Dr. T. H. Gronwall. (Abstract No. 36–3–194–*t*.)

ARNOLD DRESDEN,
Associate Secretary

THE APRIL MEETING AT STANFORD

The two hundred seventy-eighth regular meeting of the Society was held at Stanford University on Saturday, April 5, 1930. The meeting was called to order at 10:30 A.M. by President Hedrick, who, with Professor M. W. Haskell, presided over the session.

The attendance included the following twenty-five members:

Bernstein, Blichfeldt, Buck, Corbin, Engstrom, R. L. Green, M. W. Haskell, E. R. Hedrick, L. M. Hoskins, Hotelling, B. W. Jones, D. N. Lehmer, Sophia Levy, Luther, W. A. Manning, Michal, W. E. Milne, F. R. Morris, Noble, Pollock, T. M. Putnam, Pauline Sperry, Stager, Uspensky, W. M. Whyburn.

A luncheon for members and their guests was held at the Stanford Union, and later in the afternoon they were taken for a ride in the region near Palo Alto.

Titles of papers read at the meeting follow. The papers of the following authors were read by title: Basoco, E. T. Bell, Ward. The paper of Clifford Bell was read by Professor Whyburn. Mr. Debely was introduced by Professor Bernstein.

1. *On certain finite sums of binomial coefficients and gamma functions*, by Dr. M. A. Basoco. (Abstract No. 36–3–204–*t*.)

2. *On Appell's decomposition of a doubly periodic function of the third kind*, by Dr. M. A. Basoco. (Abstract No. 36–3–205–*t*.)

3. *A practical method for computing the modular representations of finite operations and relations*, by Professor B. A. Bernstein and Mr. Nemo Debely. (Abstract No. 36–3–214.)

4. *A correspondence between irregular fields*, by Professor E. T. Bell. (Abstract No. 36–5–217–*t*.)

5. *The real unit segment as a number field*, by Professor E. T. Bell. (Abstract No. 36–5–218–*t*.)

6. *Numbers of representations in certain senary quadratic forms*, by Professor E. T. Bell. (Abstract No. 36–5–219–*t*.)

7. *Analytic functions in the irregular field of all numerical functions*, by Professor E. T. Bell. (Abstract No. 36–5–220–*t*.)

8. *On a pair of simultaneous second-order differential equations*, by Dr. Clifford Bell. (Abstract No. 36–3–212.)

9. *On generalizing trigonometric identities in arithmetical paraphrasing*, by Dr. H. T. Engstrom (National Research Fellow). (Abstract No. 36–3–210.)

10. *Certain quinary forms related to the sum of five squares*, by Dr. B. W. Jones (National Research Fellow). (Abstract No. 36–3–208.)

11. *An operation that generates scalar differential invariants from tensors*, by Professor A. D. Michal. (Abstract No. 36–3–209.)

12. *The numerical determination of characteristic numbers*, by Professor W. E. Milne. (Abstract No. 36–3–206.)

13. *Simultaneous quadratic and linear representation*, by Dr. Gordon Pall (National Research Fellow). (Abstract No. 36–3–211.)

14. *An apparatus for the exhibition and study of skew curves*, by Mr. S. A. Pollock. (Abstract No. 36–3–207.)

15. *On the reduction of the indefinite binary quadratic forms*, by Professor J. V. Uspensky. (Abstract No. 36–5–221.)

16. *The algebra of recurring series*, by Professor Morgan Ward. (Abstract No. 36–3–215–*t*.)

17. *Postulates for the inverse operations in a group*, by Professor Morgan Ward. (Abstract No. 36–3–216–*t*.)

18. *On systems of cyclically related differential equations*, by Professor W. M. Whyburn. (Abstract No. 36–3–213.)

T. M. PUTNAM,
Associate Secretary

THE APRIL MEETING IN NEW YORK

The two hundred seventy-ninth meeting of the American Mathematical Society was held at Columbia University, on Friday and Saturday, April 18–19, 1930. The attendance included the following one hundred twenty-four members of the Society:

Adshead, Agnew, A. A. Albert, R. L. Anderson, R. G. Archibald, J. W. Arnold, Ball, Borofsky, Brinkmann, A. B. Brown, B. L. Brown, Carlitz, Alonzo Church, J. A. Clark, L. W. Cohen, Cooley, Cope, C. C. Craig, Cramblet, Curry, Darkow, H. T. Davis, Dehn, Demos, Eisenhart, Farnum, Feld, D. A. Flanders, M. C. Foster, Frink, Fry, Gale, C. A. Garabedian, H. L. Garabedian, Gill, Gordon, M. C. Graustein, W. C. Graustein, C. H. Graves, Gray, Gronwall, C. C. Grove, Guggenbühl, E. S. Hammond, Hille, Himwich, Hofmann, Hollcroft, R. L. Jackson, Jeffery, Joffe, Kasner, Kholodovsky, B. F. Kimball, Kline, Mark Kormes, Kramer, Lamson, Langman, Lefschetz, Lehr, Libman, Littauer, N. H. McCoy, MacColl, F. H. Miller, H. H. Mitchell, Molina, Monsky, T. W. Moore, Marston Morse, Mullins, C. A. Nelson, Newman, F. S. Nowlan, Ore, F. W. Owens, H. B. Owens, Pell-Wheeler, Pfeiffer, Phalen, C. S. Porter, Post, Raudenbush, Reddick, Ritt, J. H. Roberts, Robin Robinson, Romig, Ruger, Schelkunoff, Seely, Serghiesco, I. M. Sheffer, Shover, Siceloff, Simons, Smail, P. A. Smith, Sosnow, Struik, Tamarkin, Tartler, J. H. Taylor, T. Y. Thomas, Tuller, Vanderslice, Veblen, Wahlert, Weida, Weinstein, Weisner, M. E. Wells, Wexler, Whitehead, Whittemore, Widder, C. E. Wilder, F. G. Williams, Winters, Woodard, J. W. Young, Margaret M. Young, Zippin.

The sessions on Friday afternoon and Saturday morning were devoted to a symposium on differential geometry. The following papers were presented: Friday, April 18, *Differential geometry in the large*, by Professor D. J. Struik, and *Invariant methods in classical differential geometry*, by Professor W. C. Graustein; Saturday, April 19, *Space structure as a boundary-value problem*, by Professor T. Y. Thomas, and *On the general concepts of differential geometry*, by Professor Oswald Veblen. On Friday afternoon, Professor Kasner presided during Professor Struik's paper, and Professor Eisenhart during Professor Graustein's; Professor J. W. Young presided on Saturday morning.

On Friday morning the Society met in two sections for the presentation of short papers. The titles and cross references

to the abstracts of the papers read at these sessions follow below; the papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 19 were read before the section of Analysis and Algebra, Dr. Fry presiding, and those numbered 20–32 before the section of Geometry and Analysis Situs, Vice-President Lefschetz presiding, relieved by Professor Hollcroft. Mr. Cady was introduced by Professor Brinkmann, Mr. Howitt by Professor Struik, and Mr. Hull by Professor Nowlan.

1. *Sets of integral elements of certain rational Dickson algebras*, by Professor F. S. Nowlan and Mr. Ralph Hull. (Abstract No. 36–5–222.)

2. *On direct products, cyclic division algebras, and pure Riemann matrices*, by Dr. A. A. Albert. (Abstract No. 36–5–223.)

3. *On Galois fields of certain types*, by Mr. Leonard Carlitz. (Abstract No. 36–5–224.)

4. *On real quadratic fields*, by Mr. Charles Wexler. (Abstract No. 36–5–225.)

5. *The uniform approximation of a summable function by means of step-functions*, by Professor R. L. Jeffery. (Abstract No. 36–5–226.)

6. *Note on point transformations and non-analytic functions*, by Professor I. M. Sheffer. (Abstract No. 36–5–227.)

7. *The homogeneous case of the linear differential equation of infinite order with polynomial coefficients*, by Professor H. T. Davis. (Abstract No. 36–5–228.)

8. *An extension of the classical algebra and calculus*, by Mr. W. M. Cady. (Abstract No. 36–5–229.)

9. *The behavior of bounds and oscillations of sequences of functions under regular transformations*, by Mr. R. P. Agnew. (Abstract No. 36–5–230.)

10. *On a class of finite sums*, by Mr. Leonard Carlitz. (Abstract No. 36–5–231.)

11. *The structure of matrices with any normal division algebra of multiplications*, by Dr. A. A. Albert. (Abstract No. 36–5–232–*t*.)

12. *Note on central motions*, by Mr. Hassler Whitney. (Abstract No. 36–5–233–*t*.)

13. *A proof of the generalized second limit-theorem in the theory of probability*, by Professor Maurice Fréchet and Dr. J. A. Shohat. (Abstract No. 36–5–234–*t*.)

14. *The eliminant in electric circuit theory*, by Mr. Nathan Howitt. (Abstract No. 36–5–235.)

15. *Manifolds of functions determined by systems of algebraic differential equations*, by Professor J. F. Ritt. (Abstract No. 36–5–236.)

16. *On uniform summability of sequences of continuous functions*, by Mr. R. P. Agnew. (Abstract No. 36–5–237–*t*.)

17. *On the relation between certain methods of summability*, by Mr. H. L. Garabedian. (Abstract No. 36–5–238–*t*.)

18. *Geometry of element transformations*, by Professor Edward Kasner. (Abstract No. 36–5–239.)

19. *The representation of projective spaces*, by Mr. J. H. C. Whitehead. (Abstract No. 36–5–240.)

20. *Note on the Gauss-Bonnet formula of differential geometry*, by Professor B. F. Kimball. (Abstract No. 36–5–241.)

21. *Theorems on rotated and inverted congruences*, by Professor Malcolm Foster. (Abstract No. 36–5–242.)

22. *Degenerate algebraic manifolds*, by Professor T. R. Hollcroft. (Abstract No. 36–5–243.)

23. *Coalescence of parts of a complex*, by Dr. A. B. Brown (National Research Fellow). (Abstract No. 36–5–244.)

24. *A note concerning cactoids*, by Dr. J. H. Roberts (National Research Fellow). (Abstract No. 36–5–245.)

25. *Sets strongly homeomorphic with the interior of a plane circle*, by Professor D. W. Woodard. (Abstract No. 36–5–246.)

26. *Conical accessibility and the topologic sphere*, by Dr. Leo Zippin. (Abstract No. 36–5–247.)

27. *Cyclicly connected continuous curves whose complementary domain boundaries are homeomorphic, preserving branch points*, by Mr. V. W. Adkisson. (Abstract No. 36–5–248–*t*.)

28. *Note on components and constituents of open sets*, by Professor R. G. Putnam. (Abstract No. 36–5–249–*t*.)

29. *Potentially regular point sets*, by Professor G. T. Whyburn, (Abstract No. 36–5–250–*t*.)

30. *Concerning the structure of regular curves*, by Professor G. T. Whyburn. (Abstract No. 36–5–251–*t*.)

31. *Concerning irreducible* ϵ*-separations, locally connected spaces, and accessible plane continua*, by Professor G. T. Whyburn. (Abstract No. 36–5–252–*t*.)

32. *Transformations of compact spaces and their fixed points*, by Professor Solomon Lefschetz. (Abstract No. 36–5–253.)

ARNOLD DRESDEN,
Associate Secretary

THE APRIL MEETING IN CHICAGO

The two hundred eightieth regular meeting of the Society was held at the University of Chicago on Friday and Saturday, April 18–19, 1930. About one hundred fifty persons attended the meeting, among whom were the following one hundred eighteen members of the Society, the largest number of members ever attending a meeting in Chicago:

Axen, W. L. Ayres, R. W. Babcock, Bardell, Barnard, Barron, Bartky, Baten, Beenken, Theodore Bennett, G. A. Bliss, Blumberg, Bowles, Boyce, O. E. Brown, L. H. Bunyan, R. D. Carmichael, Coble, Coe, A. H. Copeland, Copp, H. B. Curtis, Curtiss, Dancer, Dickson, Doak, Duren, Dresden, Earl, Edington, Emch, Escott, H. P. Evans, Everett, Feltges, Finan, Focke, Georges, J. D. Grant, L. M. Graves, Griffiths, V. G. Grove, W. L. Hart, Hartung, Hildebrandt, Hodge, Charles Hopkins, J. C. Hughes, Ingraham, Dunham Jackson, W. S. Kimball, Kinney, Krathwohl, E. P. Lane, Langer, LaPaz, Larsen, Latimer, LeStourgeon, Harry Levy, Logsdon, Lunn, McShane, MacDuffee, MacMillan, Maier, March, T. E. Mason, B. I. Miller, E. H. Moore, Max Morris, E. J. Moulton, Musselman, Nyswander, E. J. Olson, H. L. Olson, Oppenheim, Palmer, Parkinson, Pennell, Pepper, O. J. Peterson, Pixley, Plant, Powell, Irene Price, Rainich, M. S. Rees, W. T. Reid, D. P. Richardson, H. L. Rietz, Ross, Roth, Rouse, Rowland, Sanger, Schottenfels, Skinner, Slaught, Sokolnikoff, Sparks, Speeker, Stafford, Swingle, Thielman, Trump, Vass, Walton, L. E. Ward, Warren Weaver, Wegner, M. J. Weiss, Wiley, K. P. Williams, Frederick Wood, F. E. Wood, Roscoe Woods, Wyant.

On Friday afternoon Professor Henry Blumberg gave a symposium address on *Methods in point sets and the theory of real functions*. Professor Bliss, who presided at this session, expressed the thanks of those present to the speaker.

On Friday evening members and their guests attended a dinner in the Hotel Windermere West. Professor R. D. Carmichael was toastmaster. He called upon Professor Hildebrandt as winner of the last Chauvenet prize. Dean Focke spoke on the coming annual meeting of the Society in Cleveland, and Professor Dresden brought the greetings of the Eastern group to this meeting. The last speaker, Professor Moore, described the development of General Analysis. One hundred eight persons attended this dinner.

The Council met on Friday afternoon.

It was announced that the following twelve persons were elected to membership:

Mr. Harold Maile Bacon, Los Angeles;
Mr. Roberts Cozart Bullock, University of North Carolina;
Mr. Laurens Earle Bush, University of North Carolina;
Mr. Stewart Scott Cairns, Yale University;
Mr. Edward Alexander Cameron, University of North Carolina;
Mr. Warren Randolph Church, University of Pennsylvania;
Dr. Gordon Pall, California Institute of Technology;
Mr. Oldrich Pokorny, University of Prague;
Dr. Robin Robinson, Dartmouth College;
Mr. James Jerry Slader, Jr., University of North Carolina.

As nominee of the University of Pennsylvania:
Mr. John Livezey Vanderslice;

As nominee of Allyn and Bacon:
Mr. Samuel Borofsky.

Under the reciprocity arrangement with the London Mathematical Society, the following three persons have become members of the Society:

Mr. Edward Travers Dixon, Cambridge;
Dr. Raghunath Purushottam Paranjpye, Calcutta;
Mr. Colin Macdonald Ross, Cambridge.

Twenty-two new applications were received by the Council.

The following appointments by the president were announced: to represent the Society at the dedication of the new campus and buildings of the University of California at Los Angeles, March 27–28, 1930, Professor E. T. Bell; to represent the Society at the Semi-Centennial Celebration of the Case School and the inauguration of William Elgin Wickenden as President of the Faculty, April 11, 1930, Professor B. F. Yanney; to represent the Society at the inauguration of Homer LeRoy Shantz as President of the University of Arizona, April 24, 1930, Professor H. B. Leonard.

Upon recommendation of the Committee on the Visiting Lecturership, Professor Wilhelm Blaschke of the University of Hamburg was appointed as Visiting Lecturer for the academic year 1930–31.

The Council received with thanks an invitation received from the College of William and Mary to hold the summer meeting of 1932 at Williamsburg.

The dates of the spring meeting at Chicago for 1931 were fixed for April 3, 4.

The president was authorized and requested to appoint a committee to plan a symposium at the meeting in Chicago in April 1931. He later appointed Professors R. D. Carmichael (Chairman), V. G. Grove, and M. H. Ingraham.

Minor changes in the by-laws were recommended to the Society for action at the next meeting.

On Friday morning Professor H. L. Rietz presided at the general session, Vice-President Lane at the section on Geometry, and Professor E. J. Moulton at the section on Analysis. Professor G. A. Bliss presided at the symposium on Friday afternoon, and Professor Coble at the Saturday morning session. The papers whose abstract numbers are followed by the letter *t* were read by title. Mr. Mendel was introduced by Professor E. P. Lane, Mr. Starcher by Professor R. D. Carmichael, Mr. Miller by Professor R. L. Wilder, and Mr. Munro by Professor J. A. Nyswander.

General Session, Friday morning.

1. *Point set theory applied to the random selection of the digits of an admissible number*, by Professor A. H. Copeland. (Abstract No. 36–5–254.)

2. *The discriminant matrix of a semi-simple algebra*, by Professor C. C. MacDuffee. (Abstract No. 36–5–255.)

3. *Integral surfaces of triads of partial differential equations of the third order*, by Professor E. P. Lane. (Abstract No. 36–3–170.)

4. *Unification of force and matter components*, by Professor G. Y. Rainich. (Abstract No. 36–3–195.)

5. *On the asymptotic solutions of ordinary differential equations, with an application to the Bessel functions of large order*, by Professor R. E. Langer. (Abstract No. 36–3–201.)

Section of Geometry and Point Sets, Friday morning.

6. *On the rational plane quintic curve with three cusps*, by Professor O. J. Peterson. (Abstract No. 36–3–202.)

7. *The dimensionality of a certain linear space in the projective differential geometry of a variety in hyperspace*, by Mr. C. W. Mendel. (Abstract No. 36–3–197.)

8. *A new characterization of the surface of Veronese*, by Mr. C. W. Mendel. (Abstract No. 36–5–256–*t*.)

9. *On algebraic surfaces which are invariant in a certain class of finite collineation groups*, by Professor Arnold Emch. (Abstract No. 36–3–198.)

10. *A generalization of the Scherrer fixed-point theorem*, by Professor W. L. Ayres. (Abstract No. 36-5-257.)

11. *A generalization of biconnex sets*, by Dr. P. M. Swingle. (Abstract No. 36–5–258.)

12. *Hypergeodesic mapping of a surface on a plane*, by Professor E. P. Lane. (Abstract No. 36–5–259–*t*.)

Section of Analysis, Friday morning.

13. *A characteristic normal form for the Euler equations of regular problems of the calculus of variations with prescribed transversality conditions*, by Dr. Lincoln LaPaz. (Abstract No. 36–5–260.)

14. *A generalization of Fourier series*, by Mr. W. O. Pennell. (Abstract No. 36–3–199.)

15. *Expansion problems associated with a system of integral equations*, by Dr. W. T. Reid (National Research Fellow). (Abstract No. 36–5–261.)

16. *On solution of the torsion problem for a polygon with re-entrant angles*, by Mr. I. S. Sokolnikoff. (Abstract No. 36–5–262.)

17. *Elementary properties of the t_r-functions*, by Dr. Wilhelm Maier. (Abstract No. 36–5–263.)

18. *Groups of permutable functions*, by Dr. L. M. Blumenthal. (Abstract No. 36–3–200–*t*.)

19. *A solution of a simple functional equation as a basis for readily obtaining the theory of elliptic functions*, by Mr. G. W. Starcher. (Abstract No. 36–3–203–*t*.)

General Session, Saturday morning.

20. *The constants of the disturbing function*, by Professor K. P. Williams. (Abstract No. 36–5–264–*t*.)

21. *Concerning subsets of a continuous curve S which lie on an arc of S*, by Mr. E. W. Miller. (Abstract No. 36–5–265.)

22. *On canonical forms of differential equations*, by Professor V. G. Grove. (Abstract No. 36–5–266.)

23. *A theorem on linear connections*, by Professor Harry Levy. (Abstract No. 36–5–267.)

24. *On curves of zero curvatures in Riemann spaces*, by Professor Harry Levy. (Abstract No. 36–5–268–*t*.)

25. *On matrices conjugate with respect to the minimum equation*, by Miss E. T. Stafford. (Abstract No. 36–5–269.)

26. *A determination of the integral domains of the complete rational matric algebra of order* 4, by Mr. E. J. Finan. (Abstract No. 36–5–270.)

27. *Amicable numbers*, by Mr. E. B. Escott. (Abstract No. 36–5–271.)

28. *The probable error of certain functions of the errors*, by Dr. W. D. Baten. (Abstract No. 36–5–272.)

29. *Systems of partial differential equations with constant coefficients*, by Mr. G. C. Munro. (Abstract No. 36–5–273.)

30. *An envelope theorem and necessary conditions for a problem of Mayer with one variable end point*, by Dr. M. G. Boyce. (Abstract No. 36–5–274.)

31. *Researches in the theory of numbers*, by Professor L. E. Dickson. (Abstract No. 36–5–275.)

32. *On certain universal and zero indefinite ternary quadratic forms*, by Mr. A. E. Ross. (Abstract No. 36–5–276–*t*.)

33. *On universal indefinite quaternary quadratic forms* $\phi = f(x, y, z) + \alpha w^2$, by Mr. A. E. Ross. (Abstract No. 36–5–277–*t*.)

M. H. INGRAHAM,
Associate Secretary

SHORTER NOTICES

Storia delle Matematiche. By Gino Loria. Volume I. Antichità; Medio Evo; Rinascimento. Torino, 1929. 497 pp.

This book is one that in an English translation would be very useful and welcome to teachers of mathematics, both in universities and in secondary schools. Its value to mathematicians will depend largely on their personal interest in the history and development of the subject; and the second volume, which will begin with the 16th century, will doubtless offer considerably more of interest to them. The present volume is in any case not just "another history of mathematics," for it is written in an original, individual, and vivid style which, combined with its general accuracy in matters of fact, assures it a place of its own in the literature of the subject.

There are 15 chapters, as follows: I, Ancient Mediterranean civilizations, pages 21–56; II, Greek mathematics in combinations with philosophy, 57–85; III, The law-givers of geometry [Euclid, Archimedes, Apollonius], 87–126; IV, The autumn of Greek geometry, 127–150; V, The mathematical work of the Greek astronomers and surveyors, 151–178; VI, The art of calculation and the science of number among the Greeks, 179–218; VII, "S.P.Q.R.," 219–235; VIII, Mathematics in Europe during the Dark Ages, 237–259; IX, The Chinese enigma, 261–295; X, At the foot of the Himalayas, 297–326; XI, The miracle of the Arabs, 327–377; XII, The Renaissance in Italy; Leonardo Fibonacci, 379–410; XIII, The Renaissance beyond the Alps, 411–441; XIV, Geometry in the service of painting, 443–454; XV, First manifestations of syncopated algebra, 455–487; Index of names, 489–497. Each chapter is followed by a bibliography of the most important original sources used in its preparation.

Opinions as to the best division of the limited space available in a book of this kind will necessarily vary with the individual taste and judgment of the author, but surely all will agree that one third of the space of such a volume as this is properly devoted to the Greeks. The Arabs occupy a somewhat larger proportionate share of the remainder than is usual in histories of mathematics, and this is also natural in view of the fact that the most recent researches have brought to light many new and valuable contributions made by the followers of Mahomet. For this reason this chapter is perhaps the most interesting in the book. The chapter on Leonardo of Pisa (Fibonacci) contains 26 pages, and it is interesting enough to justify the use of so much space, even apart from the fact that Leonardo is the first European writer to set out in orderly form the chief mathematical attainments of the middle ages, especially those of the Arabs, and that he appears as a star of the first magnitude shining among the lesser lights of his time in Europe.

Loria points out the great merit of the work of Levi Ben Gerson and of other Hebrews, who have perhaps been inadequately treated by most writers on the history of mathematics. To be sure, the author goes too far when he suggests calling the Sine Law of plane trigonometry "Theorem of Levi

Ben Gerson" for it was known to Albiruni* and doubtless to other Arabs before his time. The accounts of Regiomontanus and Pacioli which conclude the volume are excellent.

Unfortunately, the quality of the paper and binding is inferior; and there are a number of misprints, among which the following are perhaps worth mentioning: p. 44, in the first equation $x-1$ should be $2x-1$; p. 124, l. 19, "semidiameters" should be "conjugate semidiameters"; p. 130, l. 14, $a+b+c$ should be $a+2b+c$; p. 136, l. 13, Thucydides' date should be "*a. C.*"; p. 309, l. 16, "Brahmagupta" should be "Bhascara"; and p. 469, l. 4, "inscribed circle" should be "circumscribed circle."

It is to be hoped that the second volume will soon be published. As will have appeared from what has been said, it is the opinion of the reviewer that this first volume is a valuable addition to the available histories of mathematics, both for its lively and clear style and for its essential reliability in the synopses of the work of the writers cited.

R. B. McClenon

Sophus Lie's Gesammelte Abhandlungen (*Samlede Avhandlinger*). Edited by Friedrich Engel and Poul Heegaard. Volume IV. *Abhandlungen zur Theorie der Differentialgleichungen*, zweite Abteilung (*Avhandlinger til Differentialligningernes Teori, anden Avdeling*), edited by Friedrich Engel. Leipzig, B. G. Teubner, and Oslo, H. Aschehoug, 1929. xii+684 pp.

The three volumes of Lie's collected memoirs which have already appeared have been reviewed briefly in this Bulletin in the order of their publication, as follows: volume III in vol. 29 (1923), pp. 367–369; volume V in vol. 31 (1925), pp. 559–560; volume VI in vol. 34 (1928), pp. 369–370. The published memoirs of Lie are being gathered together in six volumes while a seventh is to be devoted to the principal works found among his literary remains. The memoirs on geometry are to go into volumes I and II, volume I being the next one promised; the memoirs on differential equations have appeared in volumes III and IV, the latter being the one now before us; while the memoirs on transformation groups are contained in the previously published volumes V and VI.

More than half of the fourth volume (pp. 1–264, with pp. 470–589 of the notes) is required for Lie's great memoirs of 1874–1877, published in the eighth, ninth and eleventh volumes of the Mathematische Annalen. These consist of a reworked and in some parts greatly extended presentation of the contents of fifteen earlier papers which are reproduced in volume III of the Gesammelte Abhandlungen.

The present volume IV contains altogether a total of eleven papers, printed on pages 1–448, while the notes and remarks (bound in a separate volume) appear on pages 449–684. Besides the three great memoirs already mentioned two other extensive papers are reproduced here, namely, the memoirs of 1895 and 1898 in the Leipziger Berichte. The editorial work continues to be done with such patience and care and skill as can be prompted only by devotion to a labor of love.

R. D. Carmichael

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* *Die trigonometrischen Lehren des persischen Astronomen . . . . . Al-Biruni*, by Carl Schoy. Hannover, 1927, p. 53.

Vector-Rechnung. By Dr. Max Lagally. Mathematik und ihre Anwendungen, II. Leipzig, Akademische Verlagsgesellschaft, 1928. xvii+358 pp. $5.50.

Punkt- und Vektor-Rechnung. By Alfred Lotze. Göschens Lehrbücherei. I Gruppe: Reine und angewandte Mathematik. Band 13. Berlin und Leipzig, Walter de Gruyter, 1929. 192 pp. Rm 12.

Vektor-Analysis. By Dr. Siegfried Valentiner. Sammlung Göschen. Berlin und Leipzig, Walter de Gruyter, 1929. Rm 1.50.

The first of these texts is the outcome of lectures covering some years, given by Dr. Lagally in the Technical High Schools of Munich and Dresden, before students who were familiar with physics and mathematics. It is therefore designed to furnish engineers with a rather complete reference book on this subject. The notation so far as possible is that of Gibbs. The book is divided into chapters as follows:

1. Elementary vector algebra, 55 pages; 2. Vectors dependent upon scalar parameters, 62 pages; 3. Theory of fields, 70 pages; 4. Dyads, 49 pages; 5. The most important dyads of mechanics, 37 pages; 6. Transformations, 25 pages; 7. Vectors in Riemann space, 47 pages; 8. Complex numbers, 15 pages.

Each chapter has several sections. The author has compressed into the book a great deal of the current methods labeled "vector." References are not too numerous but sufficient for the purpose. The author has arrived at dyads, or linear vector operators, early in the book, which he considers a desirable thing to do. He tries to give a brief exposition of quaternions and hypernumbers at the end, with ill success. The last chapter could have been omitted.

The claim is made in the preface that the approach is direct and intuitive, but when one finds that this approach is limited to a few diagrams and that the old familiar coordinate system is on the scene rather promptly, and from then on really dominates the development, the vector system sinking as usual to the mere shorthand expression for various coordinate forms, he is not ready to accede to the claim of an intuitive development.

The author recognizes the slow spread of vector methods in current literature, but he does not seem to realize that this slow spread is due to the persistent use of coordinates in at least the thought of those who would use the vector methods. There is not the slightest necessity to mention coordinates or the "Dreibein" (which may be translated *trisceles*, meaning trirectangular axes). Formulas for rotation of the trisceles and other transformations are not needed, and the invariantive character of the expressions is obvious. It seems strange that after these hundred years of ideas of Hamilton and Grassmann anyone should still be clinging to the antiquated methods and forms of coordinates. Anyone familiar with the literature and the discussions about vectors for the last forty years knows that vectors properly developed ("autonomously" in the language of Burali-Forti and Marcolongo) furnish an invariantive mode of thought and writing for geometry and physics. The mere fact that the laws of physics can be so stated shows them as invariants. This simplicity is destroyed as soon as the vectors are made to depend upon axes or coordinates. The only way such dependence should enter is in the representation of variable points, as for instance the vector ρ may depend upon a

single parameter when the terminal point P travels on a curve, or on two parameters when it is confined to a surface. Three parameters, like x, y, z, would make it variable in three-dimensional space. For differential purposes these may sometimes be useful, though even under such conditions the free differentiation in any desired direction is much better. For curves the intrinsic vectors, related to the curve, are the only ones that furnish proper directions for differentiation and the same may be said of surfaces. When this is once recognized the clumsy machinery of axes, even when oiled with vector treatment, disappears.

These remarks apply of course to scalar fields and vector fields. It shows strongly in the use of the Hamilton ∇. The present author drags this in with i, j, k attachments, a most common but unnatural method, one which makes it look artificial, and which demands invariant proofs at once. This is of course unnecessary, since the operator may be developed with no reference to coordinate axes, and has been so developed. Further the usual difficulties in numerous formulas appear with the use of the div, rot, grad notation. We cannot enlarge upon this point; it has been discussed before in many papers.

The chapter on transformations is preparatory to that on Riemann space, which follows. It contains a treatment of the "fundamental form" for a three-dimensional space as viewed from the parameter stand-point, but developed as if the ideas were still those of coordinates. It amounts to a vector treatment of curvilinear coordinates. In the following chapter we are led into space of N dimensions, with the notion of curvilinear coordinates still prominent. This is the current method, but it is not necessary to bring in vectors if one is going to do it this way. The criticism that nothing has ever been done by vectors which had not been already done some other way, is perfectly valid when vectors are brought out in this fashion; for the vector method really is nothing more than a shorthand expression of what had already been worked out in longhand.

The text is well printed, and for those who desire knowledge of what is going on in most work related to this subject it is satisfactory. It will not, however, be the starting point for further developments of vector methods.

The second book mentioned above is a rather straightforward treatment of the Grassmann Ausdehnungslehre, and the author considers Grassmann's work as the only existing basis of unification of all vector methods. The attempt is made to produce a vector analysis and a dyadic analysis, but the success is not manifest. The applications are mostly from projective geometry, and for the very good reason that the Grassmann methods are definitely projective. They do not lend themselves to metric treatment readily and it is a mistake to try to get metric results and theorems that way. A small chapter is given on mechanics, and shows at once the limitations of the method. As an introduction to the ideas of Grassmann the book will be fairly successful.

The last book mentioned is merely a reprint of the third edition, the only noticeable change being a table of formulas at the end. It is a brief introduction to the subject as expressed in the current German notation. One small chapter on applications to physics discusses three topics. For a brief glance at vectors it is a good text.

J. B. SHAW

NOTES

A new journal of interest to workers in applied mathematics, Ingenieur-Archiv, has been started, under the editorship of R. Grammel, with the assistance of A. Betz, A. Hertwig, and K. von Sanden. It is published by Springer.

The American Statistical Association has begun publication of a new journal called The Annals of Mathematical Statistics. Professor H. C. Carver, of the University of Michigan, is editor.

The British Association for the Advancement of Science will meet at Bristol, September 3–10, 1930, when Sir Thomas Holland will be succeeded in the presidential chair by Professor F. O. Bower. Dr. F. E. Smith is president of Section A (mathematical and physical sciences).

The first Congress of Mathematicians of the Slavic Countries was held at Warsaw, September 23–27, 1929, under the presidency of Professor W. Sierpinski. There was an attendance of one hundred eighty persons, including representatives from Bulgaria, Czecho-Slovakia, Jugoslavia, and Poland; Russia was not represented, the Academy of Sciences of Leningrad having declined an invitation to take part. Eight general lectures and eighty shorter papers were presented.

The Accademia Pontificia dei Nuovi Lincei has awarded its prize for a critical study of the quantum theory as follows: one half to G. Wataghin of Turin, and one fourth each to P. Straneo, of Genoa, and A. Carrelli, of Naples.

The Academy of Sciences of Turin has awarded its Vallauri prize for the period 1923–26 jointly to Alessandro Marchetti, for his work as director of the Società Idrovolanti Alta Italia, and Professor Enrico Pistolesi, of the Engineering School of Pisa, for his work in aerodynamics.

The Franklin Institute has awarded a Franklin medal to Sir William Bragg, director of the Davy-Faraday Research Laboratory of the Royal Institution of Great Britain, in recognition of his contributions to the knowledge of atomic structure.

Dr. Irving Langmuir, of the General Electric Company, has been awarded the Willard Gibbs gold medal of the Chicago Section of the American Chemical Society "for fundamental work on atomic hydrogen, on surface relations, and electrical discharge phenomena, for contributions to various branches of physical chemistry, and for his presentation of a theory of atomic structure."

The Society of Arts and Sciences has awarded one of its gold medals to Professor G. N. Lewis, of the department of physical chemistry of the University of California.

Professor T. von Kármán has received an honorary doctorate from the Berlin Technical School.

Professor Tullio Levi-Civita has been elected a corresponding member of the Prussian Academy of Sciences.

Professor C. V. Raman, of the University of Calcutta, has received an honorary doctorate from the University of Freiburg.

Mr. P. A. M. Dirac, of Cambridge, has been recommended by the Council of the Royal Society of London for election to membership in that Society.

Dr. L. Eckhart, of the Vienna Technical School, has been promoted to a professorship of descriptive geometry.

Dr. F. L. Krames, of the Vienna Technical School, has been appointed to an associate professorship of descriptive geometry at the German Technical School of Brünn.

E. Esclangon, director of the Paris Observatory, has been appointed professor of astronomy at the University of Paris, as successor to the late Professor H. Andoyer.

Professor Arnold Eucken, of the Breslau Technical School, has been appointed professor of physical chemistry at the University of Göttingen.

Professor Helmut Hasse, of the University of Halle, has been appointed professor of mathematics at the University of Marburg.

The Rockefeller Institute has given funds to endow a new Institute of Mathematics at the University of Göttingen. It will be directed by Professor R. Courant.

E. B. Schieldrop, of Drontheim, has been appointed professor of applied mathematics at the University of Oslo, as successor to the late Richard Birkeland.

Professor A. C. Banerji, of the department of mathematics in the University of Allahabad, has been promoted to the University chair of mathematics.

Dr. P. L. Srivastava has been promoted to a readership in mathematics in the University of Allahabad.

Dr. M. Fekete, of the University of Budapest, has been appointed to an associate professorship in the Einstein Institute at the University of Jerusalem.

The following have been promoted to full professorships in Italian universities: E. Fermi, to the chair of theoretical physics in the University of Rome; E. Persico, to the chair of theoretical physics in the University of Florence; R. Serini, to the chair of mathematical physics in the University of Pavia.

Professor G. Albanese, of the chair of projective and descriptive geometry at the University of Palermo, has been transferred to a professorship at the University of Pisa.

Professor G. Giorgi, of the chair of mathematical physics at the University of Cagliari, has been transferred to a professorship at the University of Palermo.

Professor A. Tonolo, of the chair of the calculus at the University of Ferrara, has been transferred to a professorship at the University of Padua.

At Cambridge, Mr. W. R. Dean, of Trinity College, has been appointed university lecturer.

The following sixty-three doctorates with mathematics or mathematical physics as major subject were conferred during 1929 in universities in the United States and Canada; the university, month in which the degree was conferred, minor subject (other than mathematics), and title of dissertation are given in each case if available.

Nola L. Anderson, Missouri, June, astronomy, *An extension of Maschke's symbolism.*

G. A. Baker, Illinois, May, economics, *Random sampling from non-homogeneous populations.*

M. A. Basoco, California Institute of Technology, June, mathematical physics, *Fourier expansions of doubly periodic functions of the third kind.*

W. D. Baten, Michigan, June, *Theorems concerning probability.*

A. C. Berry, Harvard, June, *Fourier representations.*

H. R. Beveridge, Illinois, May, astronomy, *An expansion problem connected with a system of partial differential equations.*

O. K. Bower, Illinois, May, physics, *Applications of an abstract existence theorem to both differential and difference equations.*

A. B. Brown, Harvard, June, *Relations between the critical points of a real analytic function of n independent variables.*

O. E. Brown, Chicago, August, *The equivalence of triples of bilinear forms.*

F. H. J. Burkett, New York University, June, physics, *Some properties of the sextic with a quadruple point.*

R. V. Churchill, Michigan, June, *On the geometry of the Riemann tensor.*

W. M. Coates, Michigan, June, engineering mechanics, *The state of stress in thin-walled pressure vessels.*

C. J. Coe, Harvard, June, *Exterior motion in the restricted problem of three bodies.*

J. B. Coleman, California, August, *Concerning the reducibility of the characteristic equation of a ternary continued fraction.*

A. J. Cook, Chicago, June, *Pairs of rectilinear congruences with generators in one-to-one correspondence.*

H. V. Craig, Wisconsin, October, *On the simultaneous differential invariants of two functions with an application to the calculus of variations.*

Mildred W. Dean, Johns Hopkins, February, *Studies in inversive geometry with reference to a special set of six points.*

L. A. V. DeCleene, Catholic, *On triangles circumscribed about a conic and inscribed in a cubic curve.*

H. P. Doole, Nebraska, June, physics, *Certain multiple-parameter expansions.*

H. H. Downing, Chicago, December, *Absolute minima for space problems of the calculus of variations in parametric form.*

Nat Edmonson, Jr., Rice, June, *Poisson's integral and plurisegments on the hypersphere.*

P. D. Edwards, Indiana, June, astronomy, *Functions possessing an addition theorem from standpoint of a functional equation given by Abel.*

J. D. Elder, California Institute of Technology, June, mathematical physics, *Arithmetized trigonometrical expansions of doubly periodic functions of the third kind.*

H. T. Engstrom, Yale, June, *On the common index divisors of an algebraic field.*

H. P. Evans, Wisconsin, June, *The two dimensional boundary value problem for the transmission of alternating currents through a heterogeneous earth.*

E. C. Goldsworthy, California, May, physics, *Curves autopolar with respect to a cyclic set of conics.*

Deborah M. Hickey, Rice, June, *A three dimensional treatment of groups of linear transformations.*

J. J. L. Hinricksen, Harvard, June, *The problem of n bodies.*

T. J. Jaramillo, Chicago, March, *A generalization of the energy function of elasticity theory.*

C. I. Lubin, Harvard, June, *Singular points of second order systems of real differential equations.*

Dorothy McCoy, Iowa, July, psychology, *The complete existential theory of eight fundamental properties of topological spaces.*

N. H. McCoy, Iowa, June, physics, *Commutation formulas in the algebra of quantum mechanics.*

Sister Marie Cecilia Mangold, Catholic, June, education, *On the loci of vertices of singly infinite systems of triangles circumscribed about a fixed conic.*

R. H. Marquis, Chicago, June, *A comparison of existing theories of algebraic numbers, and the isolation of an abstract theory.*

H. A. Meyer, Iowa, June, physics, *On certain inequalities with applications in actuarial science.*

E. L. Mickelson, Minnesota, July, physics, *On the approximate representation of a function of two variables.*

E. R. C. Miles, Rice, June, (1) *Boundary value problems for potentials of a single layer in the plane.* (2) *Potentials of general masses in single and double layers. The relative boundary value problems.*

H. J. Miles, California, May, astronomy, *On a generalization of Plücker's surface.*

A. K. Mitchell, Johns Hopkins, June, *The derivation of tensors from tensor functions.*

W. K. Morrill, Johns Hopkins, June, *A problem of ambiency.*

Max Morris, Chicago, June, *Nets of collineations in n-space.*

E. D. Mouzon, Illinois, January, economics, *Equi-modal frequency distributions.*

C. F. Muckenhaupt, Massachusetts Institute of Technology, June, physics, *Almost periodic functions and vibrating systems.*

C. O. Oakley, Illinois, May, physics, *Differential equations containing absolute values of derivatives.*

Gordon Pall, Chicago, June, *Problems in additive theory of numbers.*

G. A. Parkinson, Wisconsin, August, *Pairs of curves in a S_n. A theory of parallelism in sub-spaces.*

F. D. Perez, Chicago, August, *The Hilbert-Schmidt theory of linear integral equations in general analysis for quaternionic-valued functions.*

J. C. Polly, Cornell, June, physics, *Rational surfaces mappable by C_{2n}: $8A^nB^{n-1}$.*

Bryan Priestman, McGill, October, major in physics, minor in mathematics, *Propagation of quanta.*

W. T. Reid, Texas, June, *Properties of solutions of an infinite system of ordinary linear differential equations of the first order with ordinary boundary conditions.*

J. H. Roberts, Texas, June, *Concerning non-dense plane continua.*

Robin Robinson, Harvard, February, *On the differential geometry of surfaces in non-euclidean space.*

E. B. Roessler, California, May, architecture, *A certain family of autopolar sextics.*

Samuel Silberfarb, Chicago, August, *Representation by indefinite ternary quadratic forms.*

C. A. Spicer, Johns Hopkins, June, *P-curves of Cremona transformations.*

L. H. Swinford, California, May, Greek, *On rational autopolar quintic curves.*

P. M. Swingle, Michigan, June, *A certain type of continuous curve and related point set.*

S. C. Wang, Columbia, major in physics, *The problem of the normal hydrogen molecule in the new quantum mechanics.*

J. H. Webb, Wisconsin, June, major in physics, minor in mathematics, *Potential distribution due to a buried sphere under the influence of a point electrode.*

Rose A. Whelan, Brown, June, *Approximate solutions of certain general types of boundary value problems from the standpoint of integral equations.*

F. G. Williams, Cornell, June, physics, *Families of plane involutions of genus 2 or 3.*

Kathryn Wyant, Missouri, August, astronomy, *The ideals in the algebra of generalized quaternions over the field of rational numbers.*

Leo Zippin, Pennsylvania, June, *A study of continuous curves and their relation to the Janiszewski-Mullikin theorem.*

Professor R. W. Babcock, head of the department of mathematics at DePauw University, has been appointed dean of the division of general science of the Kansas State Agricultural College.

Assistant Professor F. R. Bamforth, of Cornell University, has been appointed to an assistant professorship of mathematics at the Ohio State University.

Professor J. W. Barker, of Lehigh University, has been appointed dean of the faculty of engineering of Columbia University.

Dr. O. E. Brown has been appointed associate professor of mathematics at Lawrence College.

Professor K. T. Compton, of the department of physics of Princeton University, has been appointed president of the Massachusetts Institute of Technology.

Dr. E. R. van Kampen, who is Professor Schouten's assistant at the University of Delft, has been appointed associate in mathematics at Johns Hopkins University.

Professor A. E. Kennelly, of the department of electrical engineering of Harvard University, retires at the close of the present academic year with the title of professor emeritus.

Dr. Lincoln LaPaz, of the University of Chicago, has been appointed assistant professor of mathematics at the Ohio State University.

Associate Professor J. H. Neelley has been promoted to a professorship of mathematics at the Carnegie Institute of Technology.

Dr. L. W. Nordheim, of the University of Göttingen, has been appointed visiting professor of theoretical physics at the Ohio State University for the spring quarter.

Dr. Tibor Rado, of the University of Szeged, Hungary, has been appointed professor of mathematics at the Ohio State University. Dr. Rado was visiting lecturer at Harvard University during the first semester of this year, and is lecturing at Rice Institute during the present semester.

Dr. T. J. J. See, who, as captain in the Navy, has been in charge of the naval chronometer and time station at Mare Island, Calif., for the last twenty-seven years, has retired from active service.

Professor H. W. Tyler, for many years professor of mathematics and head of the department of mathematics at the Massachusetts Institute of Technology, will retire at the end of the present academic year. He will continue to act as General Secretary of the American Association of University Professors, and will be also a consultant to the Library of Congress on scientific literature. Professor F. S. Woods has been appointed as head of the department of mathematics.

Professor C. A. Wheeler, of the Connecticut Agricultural College, has retired.

Dr. A. Wintner, who has been Professor Lichtenstein's assistant at the University of Leipzig, and is at present a Rockefeller Fellow at Rome, has been appointed associate in mathematics at the Johns Hopkins University.

Professor G. S. Chowla, of the Government College, Lahore, died in December, 1929.

Professor Onorato Niccoletti, of the University of Pisa, died December 31, 1929, at the age of fifty-nine.

Mr. F. P. Ramsey, lecturer in mathematics at King's College, Cambridge, died January 19, 1930, at the age of twenty-six.

Carl Schoch, known as a student of ancient astronomical records, died November 19, 1929.

Professor W. O. Beal, of the department of astronomy of the University of Minnesota, died February 15, 1930, at the age of fifty-six. Professor Beal had been a member of the American Mathematical Society since 1914.

Professor J. B. Johnson, head of the department of mathematics at Baylor University, died December 18, 1929. Professor Johnson was a member of the American Mathematical Society.

Dr. Christine Ladd-Franklin, lecturer in psychology and logic at Columbia University, died March 5, 1930, in her eighty-third year. Mrs. Ladd-Franklin held a doctorate in mathematics from Johns Hopkins University.

Professor G. W. McCoard, of the department of mathematics at the Ohio State University, died March 19, 1930, at the age of eighty.

Mr. R. E. Peterson, of Pennsylvania State College, died in the summer of 1929. Mr. Peterson was a member of the American Mathematical Society.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume.* Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

217. Professor E. T. Bell: *A correspondence between irregular fields.*

Dickson extended the theory of numbers by a correspondence between fields (this Bulletin, vol. 23 (1916), p. 109). Correspondence is here defined for irregular fields, and is applied to such fields whose elements are respectively the (infinite) set of all numerical functions, and a certain infinity of power series with radius of convergence unity. The regular elements (those having reciprocals) in the second irregular field are those power series of the kind defined with constant term different from zero. The simplest example of multiplication in this irregular field is the process by which, when legitimate, a Lambert series is derived from a given power series. Examples can be constructed showing that the restriction to the unit circle as the region of convergence is necessary as well as sufficient. (Received April 5, 1930.)

218. Professor E. T. Bell: *The real unit segment as a number field.*

In a general discussion of the theory of unique decomposition (multiplicative, additive, etc.) into indecomposable elements, it becomes necessary to devise an algebra, arithmetic and analysis for real vectors with $n>3$ components. This problem can be reduced to the situation described in the title. The set of all real numbers in the closed interval $[-\infty,+\infty]$ is put in (1,1) correspondence with $[0,1]$, so that $-\infty, 0, 1, +\infty$ correspond respectively to $0, 1/2, 3/4, 1$, and the rational operations in $[0, 1]$ are then defined in abstract identity with those for $[-\infty,+\infty]$ as in analysis. Order relations are introduced in an obvious manner. To the rational integer m in $[-\infty,+\infty]$, corresponds in $[0, 1]$ $m'=1/2 + (1/\pi)\tan^{-1} m$, $|\tan^{-1} m| \leqq \pi/2$, and the "rational integer" m' in $[0, 1]$ is "divisible arithmetically" by the rational integer n' in $[0, 1]$ when and only when m is arithmetically divisible by n in $[-\infty, +\infty]$. (Received April 5, 1930.)

219. Professor E. T. Bell: *Numbers of representations in certain senary quadratic forms.*

*See pp. 1–2 and p. 45 of the January issue.

The form $x^2+ay^2+bz^2+cw^2+du^2+et^2$ is denoted by $(1, a, b, c, d, e)$. It is shown that the number of representations of any integer n in each of the forms $(1, 1, 1, 1, 1, 4)$, $(1, 1, 1, 1, 4, 4)$, $(1, 1, 1, 4, 4, 4)$, $(1, 1, 4, 4, 4, 4)$, $(1, 4, 4, 4, 4, 4)$, is a polynomial in the real divisors of n alone, with the single exception of the case $n \equiv 1$, (mod 4). If $n \equiv 1$, (mod 4), then the number of representations is a polynomial in the divisors of n plus a numerical constant times the number of representations of n in the form $(1, 1, 1, 1, 1, 16)$. The author has not succeeded in proving whether or not the last is reducible to a divisor function. (Received April 5, 1930.)

220. Professor E. T. Bell: *Analytic functions in the irregular field of all numerical functions.*

An irregular field ϕ differs from an abstract field F only in the presence in ϕ of an infinity of elements having no reciprocal in ϕ, instead of the unique such element zero in F. In the author's *Algebraic Arithmetic* (Colloquium Publications of this Society, vol. 7, 1927) was summarized his previous work on the algebraic aspects of the irregular field of all numerical functions. The algebraic theory there developed is complete. In this paper the theory is extended to essentially infinite processes in the same irregular field, and a complete theory of such is given. As an interesting detail, the functional equations for the elementary functions and for the theta and elliptic functions have, in the irregular field, solutions reducible to simple closed forms. The formal properties of the new functions are abstractly identical with those of the corresponding analytic functions in the real or complex domain. (Received April 5, 1930.)

221. Professor J. V. Uspensky: *On the reduction of the indefinite binary quadratic forms.*

According to Hermite the reduction theory of indefinite binary quadratic forms can be based on consideration of integers giving successive minima of a positive form $\xi^2+\lambda\eta^2$ where λ is a variable positive parameter and ξ and η represent the conjugate linear factors of an indefinite binary quadratic form. However, the reduced forms obtained by this method differ from those of Gauss. In order to obtain Gauss' reduced forms one must consider a non-homogeneous function $F=|\xi/\eta|+\lambda|\eta|$ and its successive minima as a substitute for Hermite's positive form. (Received April 5, 1930.)

222. Professor F. S. Nowlan and Mr. Ralph Hull: *Sets of integral elements of certain rational Dickson algebras.*

Sets of integral elements are determined which belong to certain rational division algebras of the type discussed in Dickson's *Algebras and their Arithmetics*, p. 66. The basal units of the algebra are $y^i x^j$, $(i, j = 0, 1, 2)$, where x satisfies the irreducible cyclic cubic $x^3-3x+1=0$, and $y^3=\delta$, with δ a rational integer. We write $\delta=\eta\epsilon$, where η has no rational prime factors other than 3 or those of the forms $9m\pm1$, and where the rational prime factors of ϵ are of the forms $9m\pm2$ and $9m\pm4$, with one factor occurring to a power which is not a multiple of 3. We add the further restriction that ϵ is itself of one of the forms $9m\pm2$ or $9m\pm4$. For each such algebra there is a unique set of integral elements which contains the original nine basal units. These original basal units do not, however, constitute a basis. (Received March 24, 1930.)

223. Dr. A. A. Albert: *On direct products, cyclic division algebras, and pure Riemann matrices.*

In this paper certain theorems on the direct products of normal simple algebras are secured, and a necessary and sufficient condition that a direct product of any two division algebras be a division algebra is found. The theory of the representation of a normal division algebra in n^2 units over F as an algebra of m-rowed square matrices with elements in F is discussed, and it is proved that such a representation is possible if and only if n^2 divides m. The theory of the structure of cyclic division algebras is studied, and the problem reduced to the case of algebras of orders p^{2e}, p a prime. Also certain necessary conditions that a cyclic algebra be a division algebra are discussed. Finally the above results are utilized to prove two extremely important results in the theory of pure Riemann matrices: *The multiplication index of a pure Riemann matrix of genus p is a divisor of $2p$; when the multiplication algebra of a pure Riemann matrix is a known normal division algebra, its order is a power of two.* (Received March 12, 1930.)

224. Mr. Leonard Carlitz: *On Galois fields of certain types.*

In this paper are deduced relationships between Galois fields of prescribed construction and some of their sub-fields. Relations between the zeta functions and the discriminants of the various fields are set up. It is not here found necessary (as in the work of Artin, Herglotz, and Pollaczek) to make use of the Hecke functional equation for the zeta function of an arbitrary field. From the zeta and discriminantal relations it is possible to express a quotient of class numbers in terms of a corresponding quotient of regulators. In a particular case (the group of the Galois field is the group of a solvable equation of prime degree) the quotient of regulators is examined and shown to lie between bounds of a simple sort. (Received March 25, 1930.)

225. Mr. Charles Wexler: *On real quadratic fields.*

By means of new and rather simple methods, suggested by computing the classes in all fields up to 1,000, the following theorems concerning real quadratic fields are proved. The norm of the fundamental unit in an infinite number of fields is equal to -1. If $D=p$ or $2p$, where $p\not\equiv 1 \pmod 4$ is prime, then the field defined by $D^{1/2}$ has an odd number of classes. A necessary and sufficient condition that such a field be simple is given. As an interesting special case, it is proved that if D is prime or twice a prime, and if it can be written in the form $n^2\pm 2$, then a necessary and sufficient condition that the field defined by $D^{1/2}$ be simple is that $D-a^2=p$ or $2p$, p a prime, for all $a<D^{1/2}$. In general, if D is composite, necessary and sufficient conditions that each genus contain but one class are given. (Received March 27, 1930.)

226. Professor R. L. Jeffery: *The uniform approximation of a summable function by means of step-functions.*

Let $f(x, y)$ be bounded and measurable on a bounded domain R and let S be a measurable set on R. We are concerned with the existence of a function $K(x, y)$ constant on each of a finite number of rectangles, and such that $\int_{S_x}|f-K|dx<\epsilon$ uniformly in y, and $\int_{S_y}|f-K|dy<\epsilon$ uniformly in x, where S_x and

S_y are the respective linear sections of S. Let F be bounded and measurable on a bounded n-dimensional domain R. Let ω_p be a cell with equal sides containing a point p of R. Let I_p^{ω} be the ratio $\int_{\omega_p} F dx : m\omega_p F(p)$. Let E be the part of R for which I_p^{ω} converges to unity as ω_p converges to zero. Then $mE = mR$, and E contains a subset $E_{\epsilon\eta}$ with $mE - mE_{\epsilon\eta} < \epsilon$ at each point of which F has saltus $< \eta$ relative to $E_{\epsilon\eta}$. It is then sufficient for the existence of $K(x, y)$ that for each η and λ there exists δ such that both $m(CE_{\epsilon\eta} \cdot S_x)$ and $m(CE_{\epsilon\eta} \cdot S_y)$ are less than λ for all x and y, provided $\epsilon < \delta$. (Received March 26, 1930.)

227. Professor I. M. Sheffer: *Note on point transformations and non-analytic functions.*

A non-analytic function $f(z)$ generates a point transformation which is in general non-(directly) conformal; angle is thus distorted. Hedrick, Ingold, and Westfall have studied this aspect, and have arrived at interesting results; they derive the "characteristic lines" and set up a classification of non-analytic functions. The present paper starts from the same hypothesis of non-conformality. To each direction dz at a point we associate a direction δz such that the angle between dz and δz is preserved under transformation. Such pairs we term *dual directions*. There is a unique pair of orthogonal dual directions at a point; these give the "characteristic lines." There is a unique pair of coincident dual directions: the *contact directions*. The function $|f(z)|$ has its extremals along the characteristic lines (as Hedrick, Ingold, and Westfall have shown) and amp $f(z)$ has its extremals in the contact directions. These directions are interpreted on the derivative circle, and the position of dual directions found. Finally, we consider derivative conditions which serve to determine $f(z)$. (Received March 24, 1930.)

228. Professor H. T. Davis: *The homogeneous case of the linear differential equation of infinite order with polynomial coefficients.*

The problem studied is that of determining conditions under which solutions exist for the differential equation $\sum_{i=0}^{\infty} A_i(z) x^i \to u(x) = 0$, where the $A_i(z)$ are functions having Taylor's expansions about the origin and z is the differential operator d/dx. Two classes of solutions are discussed: (1) those of bounded degree (Stufe) and (2) those of unbounded degree. By degree we mean the limit $L = \lim_{n \to \infty} |u^{(n)}|^{1/n}$. The Pincherle-Bourlet generatrix equation is made central to the discussion and the solutions appear in the form of Laplace integrals. In the bounded class the theorem of Perron (Mathematische Annalen, vol. 84 (1921), pp. 31–42) is derived. In the unbounded class Borel summability is employed and an extension of Cauchy's integral formula for the case of fractional index is found useful. When $p = 1$, the regions of integration of the Laplace integrals are discussed for the cases (1) where the positive powers term of the Laurent expansion of $A_0(z)/A_1(z)$ in the outer annulus is algebraic, and (2) where it is transcendental. (Received March 20, 1930.)

229. Mr. W. M. Cady: *An extension of the classical algebra and calculus.*

A generalization of the ordinary modes of combination of complex numbers, viz. addition, multiplication, and exponentiation, leads to a denumerable infinity of other modes. The algebra embraced by any two consecutive modes is commutative, associative, and distributive. In the present paper the modes above exponentiation are investigated; certain difficulties with the modes below addition have prevented work in that direction. It is shown that all numbers (with certain deletions) form a group for any mode of combination, and that these groups are isomorphic to each other. An extension of the algebras thus developed leads to a generalization of the integral calculus, whereby all the elements of a continuum may be combined under any mode. In particular, the calculus corresponding to multiplication is discussed, and certain theorems are proved for it which are analogous to theorems in the ordinary calculus. (Received March 27, 1930.)

230. Mr. R. P. Agnew: *The behavior of bounds and oscillations of sequences of functions under regular transformations.*

In this paper is discussed the transformation of sequences of functions by regular linear transformations with triangular matrices whose application to sequences of constants has been extensively treated. The first chapter is on the behavior of ultimate bounds of real sequences under real transformations. The second and third chapters consider continuous oscillation, continuous convergence, uniform oscillation, and uniform convergence of complex and real sequences under complex and real transformations. In each case, bounds, oscillations, and convergence are considered (1) over the set as a whole, (2) at a single point or limit point of a set, and (3) at all points and limit points of a set. The fourth chapter is on the behavior of mean-square oscillation and convergence in the mean of complex and real sequences under complex and real transformations. (Received March 25, 1930.)

231. Mr. Leonard Carlitz: *On a class of finite sums.*

Generalizing a well known method for the evaluation of $\sum_{i=1}^{m} i^n$ we show in this paper how to sum $\sum_{i=1}^{m} [i(i-1) \cdots (i-\lambda+1)]^n$. The general term is expressed as a sum of factorials. The coefficients involved form arrays of numbers that can be calculated from a recursion formula; for $\lambda=1$ the array is the array of Stirling numbers. The expression for the sum contains $\lambda(n-1)+1$ terms. When λ is even, the method may be modified in such a way as to reduce the number of terms to approximately one-half. (Received March 25, 1930.)

232. Dr. A. A. Albert: *The structure of matrices with any normal division algebra of multiplications.*

Let B, a normal division algebra of n^2 units over F, contain a quantity a of grade n with respect to F and $\alpha_1, \cdots, \alpha_n$ as scalar roots of its minimum equation. Suppose that m is an integer divisible by n^2 so that B is representable by an algebra of m-rowed square matrices with elements in F and in particular

$m=nm'$. The author has proved the existence of certain n-matrices P_j which are m'-rowed square non-singular matrices with elements in $F(\alpha_1, \cdots, \alpha_n)$, and that when F is a field of complex numbers and ω a complex matrix with $p'n$ rows and m columns, the matrix ω has B as an algebra of multiplications if and only if ω is isomorphic in F to $\| tP_j\alpha_j^{k-1} \|$, $(j,\ k=1, \cdots, n)$, where t has p' rows and m' columns. This result is applied to give a complete determination of the structure of any pure Riemann matrix. (Received March 12, 1930.)

233. Mr. Hassler Whitney: *Note on central motions.*

Birkhoff (Göttinger Nachrichten, 1926, pp. 81–92) states that in the two-dimensional analytic case the second set of non-wandering motions M_2 is identical with the central motions M_r. This note proves the same conclusion for the case of a one-to-one continuous transformation on the surface of the sphere. (Received March 22, 1930.)

234. Professor Maurice Fréchet and Dr. J. A. Shohat: *A proof of the generalized second limit-theorem in the theory of probability.*

The above-mentioned proof is based on the following lemma. Given a sequence of laws of total probability $F_n(x)$ $(n=1, 2, \cdots)$ having moments of all orders. Assume that the moments of order k of this sequence are uniformly bounded for any given $k=0, 1, 2, \cdots$. Then at least one subsequence $F_{n_p}(x)$ exists which approaches a limiting law of total probability $F(x)$ for any x in $(-\infty, \infty)$, so that the kth moment of $F_{n_p}(x)$ approaches the kth moment of $F(x)$ $(k=0, 1, 2, \cdots)$, Two features, it seems, deserve attention. First, the generality of the statement, including the classical theorem as a very special case. Second, the method of proof, which uses neither characteristic functions nor continued fractions, but is based on a well known theorem on monotonic functions. (Received March 28, 1930.)

235. Mr. Nathan Howitt: *The eliminant in electric circuit theory.*

The Kirchoff equations of the electric circuit can be obtained from the inductance, capacity, and resistance quadratic forms and Lagrange's equations. The impedance function is the quotient of the determinant of the coefficients of the currents and the minor of the element in the first row and first column, and for a finite network is the quotient of two polynomials. The coefficients of these polynomials are obtained directly from the matrices of the coefficients of the above quadratic forms. The conditions for the preservation of the form of the impedance function are given in terms of the eliminant of these two polynomials. Short circuit corresponds to the vanishing of the eliminant. By removal of elements of the network subject to the above conditions, the so-called canonical forms, obtained by Foster by partial fraction expansion and by Cauer by continued fraction expansion, result. The conditions that a function having the form of an impedance function be an impedance function of a physical network are given in terms of the eliminant, and the equivalence equations for the invariance of the impedance function are generalized for n meshes. (Received April 14, 1930.)

236. Professor J. F. Ritt: *Manifolds of functions determined by systems of algebraic differential equations.*

We consider any system of algebraic differential equations in one independent variable x, and in n dependent variables $y_1, \cdots, y_n$, the coefficients in the equations being analytic functions belonging to a given field. A system is called irreducible if, given two polynomials G and H in the y's and their derivatives, with coefficients in the given field, such that GH vanishes for all solutions of the system, then either G vanishes for all solutions or H does. It is shown that every system is equivalent to a finite number of irreducible systems. The structure of an irreducible system is then investigated. A single equation, called the resolvent of the irreducible system, is found, in terms of the general solution of which the complete solution of the irreducible system can be expressed by rational processes. The results obtained create a perfect analogy between the notion of the complete solution of a system of algebraic differential equations and the notion of algebraic function of several variables. (Received April 17, 1930.)

237. Mr. R. P. Agnew: *On uniform summability of sequences of continuous functions.*

It has been shown by D. C. Gillespie and W. A. Hurwitz that any bounded real sequence $\{s_n(x)\}$ of continuous functions, defined over a closed compact set in metric space, which converges to a continuous function $s(x)$, is uniformly summable to $s(x)$ by a regular transformation. The transformations used were transformations with square matrices which were constructed in terms of the elements of the $\{s_n(x)\}$ sequences considered; hence different transformations were used to effect the uniform summability of different sequences. In the present note it is shown that no one regular transformation of a very general type will suffice for the uniform summability of all sequences of continuous functions which converge over an infinite set to a continuous limit function. (Received March 31, 1930.)

238. Mr. H. L. Garabedian: *On the relation between certain methods of summability.*

In this paper a comparison is made between Cesàro's definition of summability and a set of definitions which are in the main of the Lindelöf type (see Lindelöf, Journal de Mathématiques, (5), vol. 9 (1903), p. 213). In particular, it is proved that every series summable (Cr), r integral and arbitrary, is also summable by the methods of LeRoy and Mittag-Leffler. Moreover, a sequence of Dirichlet's series definitions which include (Cr) summability is exhibited. (Received April 8, 1930.)

239. Professor Edward Kasner: *Geometry of element transformations.*

In the American Journal of Mathematics for 1910, the author discussed arbitrary transformations of differential elements, with reference to the number of unions which are turned into unions. For lineal elements in the plane (x, y, p), where p is the slope, ∞^2 unions are preserved. If there are more we have necessarily a contact transformation. In the present paper are studied

curves which are termed *isogonal series*, such a series being defined as ∞^1 elements which make a constant angle α with the curve obtained by joining their points. (Dually we may also define equitangential series.) In this way ∞^2 curves are obtained, ∞^2 for each value of α. For a given element (x, y, p) there are ∞^1 such curves, and it is proved that the center of curvature varies in homographic correspondence with $\tan \alpha$. The differential equation of the triply infinite family is obtained, and some geometric consequences are stated. Extensions to lineal and surface elements in space are much more complicated. (Received April 18, 1930.)

240. Mr. J. H. C. Whitehead: *The representation of projective spaces.*

This paper deals with the representation of a projective space of paths P_n, by means of an affine space, a_{n+1}, the projective connection being regarded as an affine connection for A_{n+1}. It is found that A_{n+1} admits a congruence of paths, defined by a vector which satisfies the equations $\xi^\alpha; \beta = \delta_\beta^\alpha$, and which defines a group of affine collineations for A_{n+1}. These paths all meet in a point, and, as in the flat case, each one represents a point of P_n. Assuming the affine theory, a geometric method of obtaining projective normal coordinates, z^i, is given, and it is shown that they are related to the affine normal coordinates for A_{n+1} by the equations $y^i = z^i$, $y^0 + 1 = e^{z^0}$, where z^0 is the normal factor. It is also shown that if there exists a transformation T of the $(n+1)$ variables $x^0, \cdots, x^n$ which carries one of two projective connections into the other, then they are also equivalent under transformations of the form $x^0 = x^0 + \log \rho(x^1, \cdots, x^n)$, $x^i = x^i(x^1, \cdots, x^n)$, though this need not necessarily be T itself. Finally a process is given by which a projective connection may be normalized to obtain a projective connection in the sense of T. Y. Thomas, and it is shown that all the results obtained are applicable to the theory of these projective connections. (Received April 18, 1930.)

241. Professor B. F. Kimball: *Note on the Gauss-Bonnet formula of differential geometry.*

The author gives a simple proof of the elegant Gauss-Bonnet formula by using surface vectors. This proof is an extension of the method used by C. E. Weatherburn to the more general case of any simply connected region containing only regular points of the surface, the boundary to be rectifiable with a continuously turning tangent. (Received March 28, 1930.)

242. Professor Malcolm Foster: *Theorems on rotated and inverted congruences.*

When the unit sphere is referred to any isothermal system, each line of a rectilinear congruence is associated with that point on the sphere at which the normal has the same direction, or with similar points on the two adjoint minimal surfaces determined by the given isothermal system. Thus a congruence is defined by the point (a, b) in which any line pierces the xy-plane of the moving axes at the associated point. This paper is concerned with those congruences derived by an inversion of the point (a, b) relative to the unit circle with center at the associated point and lying in the tangent plane, or those congruences derived by a rotation of (a, b) about the corresponding normal. (Received March 24, 1930.)

243. Professor T. R. Hollcroft: *Degenerate algebraic manifolds.*

The characteristics of a manifold M of order n and dimension r, M consisting of α manifolds of dimension r and orders n_i such that $\Sigma n_i = n$, are found. It is shown that the intersections of the component manifolds of M of dimension r taken j at a time, $2 \leqq j \leqq r+1$, are j-fold $(r-j+1)$-dimensional manifolds on M and on all singular manifolds of M of higher dimension than the one specified, and play the same role on M as proper multiple manifolds on a proper manifold. Bertini proved that the genus of a degenerate plane curve consisting of α components of genera p_i is $\Sigma p_i - \alpha + 1$. This is shown to hold for curves in any space and similar relations are found for surfaces and three-dimensional manifolds in any space, which leads to the following general theorem: If a degenerate manifold M of dimension r in k dimensions, $k \geqq r+1$, consists of α non-degenerate manifolds, each of dimension r and genus p_i, $i = 1, 2, \cdots, \alpha$, the genus p of M is given by the formula $p = \Sigma p_i \pm (\alpha - 1)$, in which the plus sign occurs when r is even and the minus sign when r is odd. (Received March 28, 1930.)

244. Dr. A. B. Brown (National Research Fellow): *Coalescence of parts of a complex.*

Relations are found involving the changes in Betti numbers that occur when a number of homeomorphic parts of a complex are made to coincide. In an earlier paper the author treated the case where two parts on unconnected complexes are made to coincide, and obtained more detailed relations which have important applications. In a special case of the latter case, formulas are obtained in the present paper by use of which we can find the Betti numbers of any one of the four complexes involved if we know the Betti numbers of the other three. (Received April 18, 1930.)

245. Dr. J. H. Roberts (National Research Fellow): *A note concerning cactoids.*

A cactoid M is a bounded continuous curve lying in space of three dimensions and such that (a) every maximal cyclic curve of M is a simple closed surface and (b) no point of M lies in a bounded complementary domain of any subcontinuum of M. Now it is known that there exists a bounded acyclic continuous curve C such that every bounded acyclic continuous curve is homeomorphic with a subset of C. G. T. Whyburn has shown that with respect to its cyclic elements every continuous curve is acyclic. Moreover, the cyclic elements of a cactoid are either points or spheres. Thus this question arises: *Does there exist a cactoid C such that every cactoid is homeomorphic with a subset of C?* The object of the present paper is to answer this question negatively. (Received March 27, 1930.)

246. Professor D. W. Woodard: *Sets strongly homeomorphic with the interior of a plane circle.*

A set M_1 is said to be strongly homeomorphic with a set M_2 provided (1) M_1 is homeomorphic with M_2 and (2) $\overline{M}_1$ is homeomorphic with $\overline{M}_2$. Necessary and sufficient conditions that a set be homeomorphic with the interior of a plane circle are established. It is shown that a characterization of the sphere

may be realized by a simple modification of one of the conditions. (Received March 28, 1930.)

247. Dr. Leo Zippin: *Conical accessibility and the topologic sphere.*

A continuous curve in euclidean three-space is said to be conically accessible from one of its complementary domains, D, provided that if K is a simple closed curve of the continuous curve and on the boundary of D, then K bounds a two-cell of D. It is proved that a continuous curve C which separates three-space into two domains of each of which it is the complete boundary, and from each of which it is conically accessible, is necessarily a topologic sphere, if it is compact, and a cylinder tree if it is non-compact. (Received March 14, 1930.)

248. Mr. V. W. Adkisson: *Cyclicly connected continuous curves whose complementary domain boundaries are homeomorphic, preserving branch points.*

This paper classifies certain cyclicly connected continuous curves in euclidean space of two dimensions. Among the curves M considered are those that contain only a finite number of simple closed curves, and are such that any two complementary domain boundaries are homeomorphic, preserving branch points. Such curves are classified according to the number and orders of the branch points on each complementary domain boundary. First a necessary and sufficient condition is established that a continuous (1, 1) correspondence of M (on a sphere S) into itself be extendable to S; then the curves M are classified. We prove also that any continuous (1, 1) correspondence between any two complementary domain boundaries of M can be extended to M, and also to S, for the curves in which each complementary domain boundary contains exactly three branch points. Finally, it is shown that for any one of the curves M any continuous (1, 1) correspondence sending the curve into itself can be extended to the sphere S. (Received March 28, 1930.)

249. Professor R. G. Putnam: *Note on components and constituents of open sets.*

In this note it is proved that a necessary and sufficient condition that the components of every open subset of an open set M should be open is that M be locally connected. It is also shown that this same condition must be satisfied when the constituents of the open subsets of M are open. A condition sufficient to insure that the constituents of the open subsets of M be open is obtained. (Received March 25, 1930.)

250. Professor G. T. Whyburn: *Potentially regular point sets.*

A separable metric space R is called *potentially regular* if for each point P of R there exists a monotone decreasing family of closed neighborhoods of P whose common part is exactly P and each of which has only a finite number of boundary points. If R is connected, it is potentially regular if and only if every two points of R can be separated by a finite number of points. Let R be any connected and potentially regular space; then there exists a biunivalued and continuous transformation T of R into a separable, metric space R^* which

is connected and regular in the Menger-Urysohn sense; furthermore, the property of a finite set to separate two given points of R is invariant under the transformation T. (Received March 12, 1930.)

251. Professor G. T. Whyburn: *Concerning the structure of regular curves.*

Let M be any regular curve (in the Menger-Urysohn sense), and let K be any subset of M which contains the (countable) set Q of all local separating points of M of order >2 and which is dense on every free arc in M; then every point p of M is a regular point of M relative to K, and, indeed, the order of p in M relative to K is equal to its true order relative to M itself. The following theorem concerning additions of regular curves is also proved: In order that the sum $M+N$ of a given regular curve M and an arbitrary regular curve N should be regular, it is necessary and sufficient that the closed envelope $\overline{H}$ of the set H of all ramification points of M should be punctiform. This latter theorem gives a solution to a problem proposed by Menger. (Received March 12, 1930.)

252. Professor G. T. Whyburn: *Concerning irreducible ϵ-separations, locally connected spaces, and accessible plane continua.*

The following theorem yields solutions to two problems proposed by Urysohn (Verhandelingen der Akademie te Amsterdam, 1927, No. 4, p. 144). In order that a continuum M be connected im kleinen it is necessary and sufficient that for each point x of M and each $\epsilon>0$, there exist an irreducible ϵ-separation of x in M. A separable, metric, and connected space R is locally connected if and only if every cutting of R between two arbitrary subsets A and B of R contains an irreducible cutting between A and B; also when and only when there exists an irreducible cutting of M between every two separated subsets A and B of R. Also a generalization of a result of Mazurkiewicz on accessible points of indecomposable continua is obtained. (Received March 12, 1930.)

253. Professor Solomon Lefschetz: *Transformations of compact spaces and their fixed points.*

In the present paper the author extends to continuous transformations of absolutely general compact metric spaces the coincidences and fixed point formulas given for manifolds in papers in volumes 28 and 29 (1926–27) of the Transactions of this Society. As an important corollary it follows that a continuous single-valued transformation of a compact metric space whose Betti numbers are those of the Hilbert parallelotope always has a fixed point. The proofs will appear as part of Chapter VII of the author's forthcoming Colloquium Lectures. (Received April 18, 1930.)

254. Professor A. H. Copeland: *Point set theory applied to the random selection of the digits of an admissible number.*

Admissible numbers enable us to answer certain questions in the theory of probability which could not be handled without some such device. An

admissible number belonging to the set $A(p)$ exhibits all of the properties of an event having the probability p. For example, the success ratio approaches p as a limit and the trials of the event are independent. In this paper we shall consider whether these properties are still exhibited if we form a new event (or new number) by selecting arbitrarily a sub-group of trials (or digits of the admissible number). Obviously this will not be the case for every such choice of the digits. However, it will be proved that in general the new number will have the same properties as the original number. (Received March 13, 1930.)

255. Professor C. C. MacDuffee: *The discriminant matrix of a semi-simple algebra.*

If $\mathfrak{A}$ is a semi-simple algebra, a basis can be so chosen that e_1 is the principal unit and the first discriminant matrix is diagonal, the diagonal elements being denoted by $g_1, g_2, \cdots, g_n$. For this normalization, the constants of multiplication c_{ijk} are in the relation $g_k c_{ijk} = g_i c_{jki}$. By using this normal form, several important properties of semi-simple algebras are established, e.g., that the first and second discriminant matrices are identical for all bases, and likewise the first and second characteristic functions. Theorems concerning the discriminant matrix of a direct sum, direct product, and complete matric algebra are also given. (Received March 25, 1930.)

256. Mr. C. W. Mendel: *A new characterization of the surface of Veronese.*

This paper considers plane curves and the so-called principal curves on surfaces immersed in five-dimensional space. It contains, among other things, a new property which characterizes the surface of Veronese, namely, that the only surface which is immersed in five-dimensional space and which has on it a two-parameter family of plane curves is the surface of Veronese. Moreover, there is given a new and simple derivation of a property which Segre proved to be characteristic of this surface, namely, that the only non-developable surface which is immersed in five-dimensional space and on which the principal curves are indeterminate is the surface of Veronese. (Received March 19, 1930.)

257. Professor W. L. Ayres: *A generalization of the Scherrer fixed-point theorem.*

It is shown that if T is a homeomorphism of the compact locally connected continuum M into a subset of itself, then M contains a cyclic element C such that $T(C)$ is a subset of C. The following two interesting results are corollaries to this theorem: If each maximal cyclic set of M is an n-dimensional simplex (n may vary for different maximal cyclic sets), then every such homeomorphism T has a fixed point. If M is a plane bounded locally connected continuum, M does not separate the plane, and T is a homeomorphism of M into a subset of itself, then T has a fixed point. The results solve a problem proposed by C. Kuratowski (Fundamenta Mathematicae, vol. 14, p. 139, footnote 2). (Received March 27, 1930.)

258. Dr. P. M. Swingle: *A generalization of biconnex sets.*

Two definitions of biconnex sets by B. Knaster and C. Kuratowski, *Sur*

les ensembles connexes (Fundamenta Mathematicae, vol. 2, pp. 214, 216) are generalized and the resulting sets are shown to be equivalent. A number of theorems are proved concerning the set thus defined and related sets. (Received March 28, 1930.)

259. Professor E. P. Lane: *Hypergeodesic mapping of a surface on a plane.*

The purpose of this paper is to study a projective analogue of the metric problem of Beltrami, to map a surface of ordinary space on a plane so that the geodesics on the surface correspond to the straight lines in the plane. The geodesics are replaced by hypergeodesics, which are defined projectively by a differential equation of the same form as the equation of the geodesics. Fubini's canonical form of the differential equations of a surface, and Wilczynski's theory of plane nets are employed to construct a general theory of this kind of hypergeodesic mapping. Then three special cases of hypergeodesic mapping are studied in some detail. Green's congruentially associated net of a given plane net is connected with the theory. A new two-parameter family of plane curves covariant to a given plane net is defined and used in a second kind of hypergeodesic mapping of a surface on a plane. (Received March 21, 1930.)

260. Dr. Lincoln La Paz: *A characteristic normal form for the Euler equations of regular problems of the calculus of variations with prescribed transversality conditions.*

The most general integrand function f for a regular problem of the calculus of variations with a prescribed (admissible) set of transversality coefficients $T_i(x, y_1, \cdots, y_n, y_1', \cdots, y_n')$, $i=1, \cdots, n$, has been determined by the writer. In the present paper a characteristic normal form for the Euler equations of such problems is obtained. It is shown that $y_k''=D_k/D$, where D is the Jacobian of the T_i with respect to the y_j' and where D_k is a determinant derived from this Jacobian by change of the kth column. In the special case in which transversality is orthogonality and in which $n=2$ the resulting normal system is reducible to the pair of equations known to be characteristic of a *natural family* (Kasner). Application is made to inverse problems of the calculus of variations. (Received March 28, 1930.)

261. Dr. W. T. Reid (National Research Fellow): *Expansion problems associated with a system of integral equations.*

This paper considers the vector integral equation $y(x)=\lambda\int_a^b K(x; s)y(s)ds+f(x)$, where $y(x)\equiv(y^{(\alpha)}(x))$ and $f(x)\equiv(f^{(\alpha)}(x))$ are vectors with n components and $K(x; s)\equiv(K_{ij}(x; s))$ is a matrix with n rows and columns. The vector method is used throughout in determining the form of the Fredholm determinant and resolvent matrix for the vector equation. It is shown that when the equation is definitely self-adjoint according to a suitable definition, the characteristic numbers are all real and infinite in number. There are established expansion theorems for certain classes of functions in terms of the characteristic solutions of the definitely self-adjoint equation. The definitely self-adjoint equation includes as a special case the symmetric system in which $K_{ij}(x; s)=K_{ji}(s; x)$, $i, j=1, 2, \cdots, n$, and the kernel matrix is closed. It

also includes as a special case the system of integral equations to which a differential system consisting of n ordinary linear differential equations of the first order together with two-point boundary conditions that is definitely self-adjoint in the sense defined by Bliss (Transactions of this Society, vol. 28 (1926), pp. 561–584) may be reduced by the introduction of the Green's matrix for the differential system. (Received March 24, 1930.)

262. Mr. I. S. Sokolnikoff: *On solution of the torsion problem for a polygon with reentrant angles.*

The solution of $\nabla^2\phi=0$ with the boundary condition $\phi=(x^2+y)^2/2$ is obtained for the T-section with an infinite flange and web. The method of solution is different from that of Trefftz and Kötter, both of whom solved the problem for an L-section, which involves only one reëntrant angle. The boundary of the T-polygon is transformed by Schwartz's transformation into the axis of reals, and the interior of the polygon into the upper half-plane. The problem is reduced to the solution of the potential problem for a half-plane with prescribed values along the axis of reals; its solution is effected by Poisson's integral in which the path of integration is transformed conformally into the real axis. The resulting integral together with the equations of transformation constitutes the solution of the problem in parametric form. The method is of sufficient theoretical generality to make it applicable to the solution of $\nabla^2\phi=0$ with prescribed values along the boundary of any rectilinear polygon. (Received March 25, 1930.)

263. Dr. Wilhelm Maier: *Elementary properties of the t_ν-functions.*

In constructing an analogue of Bernoullian polynomials corresponding to the imaginary quadratic field, we are led to the following expressions:

$$\lim_{n\to\infty} \sum_{h,k}^{0<h^2+k^2<n} e^{2\pi i(xh+yk)}/(\omega_1 h+\omega_2 k)^\nu = t_\nu(x,y) = t_\nu,$$

where the "periods" ω_1, ω_2 do not vanish together, and $0<|\Im(\omega_2/\omega_1)|$; $0<x, y<1$, $\nu=1, 2, \cdots$. There are non-linear recursion formulas by which the whole set depends rationally on t_1, t_2, t_3, and a cubic identity which connects t_1, t_2, t_3, so that the study of these functions reduces to studying t_1, t_2. If $m, n=1, 2, \cdots$; $\alpha, \beta=0, \cdots, n-1$; $0<\alpha+\beta$; $t_n(0, 0)=\tau_n$; there exists an equation of degree n^2, whose coefficients a_ν are rational in τ_4, τ_6, t_m, so that $\sum_{\nu=0}^{n^2} a_\nu \tau_n^{n^2-\nu} t_m^{\nu}(x\alpha/n, y\beta/n)=0$. Since $t_n(x, y)=t_n(x+1, y)=t_n(x, y+1)$, we are enabled to express elliptic functions rationally in terms of t_ν-functions. For instance, as pointed out by Siegel, $t_1^2+2t_2=p(\omega_1 y-\omega_2 x)$. All the theory of elliptic functions and, in particular, the algebraic relations between functions of divided argument as given by N. Abel may be based on the present theory. (Received March 10, 1930.)

264. Professor K. P. Williams: *The constants of the disturbing function.*

A systematic development by means of contour integrals is given for the constants $c_n^{(i,j)}$ that occur in Newcomb's development of the disturbing function. Recursion relations are obtained that both are very accurate and

rapid to use, and greatly lessen the labor of calculation. The few constants that are obtained directly are given by rapidly conv rging series similar to those given by Brown. (Received March 27, 1930.)

265. Mr. E. W. Miller: *Concerning subsets of a continuous curve S which lie on an arc of S.*

Conditions that a closed, bounded subset of the euclidean plane lie on an arc of the plane are due to R. L. Moore and J. R. Kline. The present paper considers the corresponding problem when the imbedding space is any continuous curve in euclidean n-space. In the course of study of the Moore-Kline conditions, it is shown that the only plane closed sets for which they represent a solution of the problem are the plane itself and the simple continuous curves. Among the results obtained is a theorem reducing the problem in euclidean n-space to the case where the continuous curve is cyclicly connected. On the basis of this theorem, the problem is solved for acyclic continuous curves, and continuous curves which are boundaries of domains complementary to plane continuous curves. For a plane continuous curve S which fails to cut the plane, it is shown that a closed totally disconnected subset F lies on an arc of S if and only if for every cyclic element, C, of S at most two components of $S-C$ contain points of F. (Received March 28, 1930.)

266. Professor V. G. Grove: *On canonical forms of differential equations.*

In this paper the author introduces a class of congruences covariantly related to the surface and conjugate to the surface, each congruence being suitable as a substitute for Fubini's projective normal congruence. Each member of the class is determined by a particular choice of an invariant R with certain properties. As an application he gives a congruence of the class mentioned above and associated with a scroll surface. (Received March 27, 1930.)

267. Professor Harry Levy: *A theorem on linear connections.*

Given, in the space characterized by the paths

$$d^2x^i/ds^2 + \Gamma^i_{\alpha\beta}(dx^\alpha/ds)(dx^\beta/ds) = 0,$$

an arbitrary k-dimensional spread V_k, there exist infinitely many coordinate systems such that the paths through the points of V_k in directions linearly dependent on any $n-k$ preassigned directions, none of which are tangent to V_k, are given by linear equations. The author finds as a special case that seems to be of great interest that in a Riemannian space there always exists a coordinate system such that the geodesics through any point of a preassigned manifold in any direction orthogonal to that manifold are given by linear equations. Applications of this theorem to simplify well known results as well as to obtain new ones are given. (Received March 28, 1930.)

268. Professor Harry Levy: *On curves of zero curvatures in Riemann spaces.*

The author shows that if the kth curvature of a curve in a Riemann space

is zero, the curve must lie on a k-dimensional spread generated by the ∞^k geodesics through a point in a direction linearly dependent on k directions, and conversely any curve contained in such a manifold has curvatures $1/\rho_m$, $m=1, 2, 3, \cdots, n-1$, which are zero for $m=k, k+1, \cdots, n-1$. It is further shown that the Frenet equations are valid for such a curve. (Received March 28, 1930.)

269. Miss E. T. Stafford: *On matrices conjugate with respect to the minimum equation.*

Matrices conjugate to a given matrix M with respect to the characteristic equation of M have been defined by Taber and Bennett for special cases and by Franklin in general. The conjugates defined by Franklin (Annals of Mathematics, vol. 23 (1921), p. 97) possess, for the characteristic equation, properties (1) and (2) here given for conjugates with respect to the minimum equation, but do not satisfy (3). The conjugates with respect to the minimum equation, as defined, have these properties: (1) conjugates are commutative as to multiplication, (2) the elementary symmetric functions of M and its conjugates coincide with the elementary symmetric functions of the roots of the minimum equation, (3) each conjugate is expressible as a polynomial in M. As a result of (1) and (2), each conjugate satisfies the minimum equation. In case the minimum equation is $(x-r)^p=0$, the conjugates form the unique set of linear polynomials in M which satisfy (1) and (2). The general conjugates are formed in an elementary manner, from conjugates of such matrices. Necessary and sufficient conditions that M and its conjugates form a basis for a linear algebra are given. (Received March 28, 1930.)

270. Mr. E. J. Finan: *A determination of the integral domains of the complete rational matric algebra of order* 4.

The domains considered are of the kind defined by Speiser which contain the principal unit. By means of two transformations which leave the discriminant invariant, any domain may be put into one of the canonical forms of which there are only a finite number for a given discriminant. All possible domains may be obtained from those in canonical form by transformations under which the set of all rational integral domains is closed. These results are an extension of those obtained by Du Pasquier. A determination of the domains for relatively small values of the discriminant is included. (Received March 18, 1930.)

271. Mr. E. B. Escott: *Amicable numbers.*

Two numbers are called amicable if each equals the sum of the aliquot divisors of the other, i.e., the sum of the divisors of the number, excluding the number itself. This can be expressed by means of the two equations $S(m)=m+n=S(n)$, where $S(m)$ is the sum of the divisors of m including 1 and m itself. Since the problem involves the question of the primality of the factors of the numbers considered, the solution requires many trials. This paper gives an original method by which the number of necessary trials is reduced very materially. The following is a list of the number of amicable pairs known at present: Iamblichus 1 (283–330 A.D.), Fermat 1 (1636), Descartes 1 (1638),

Euler 59 (1750), Legendre 1 (1830), Paganini 1 (1866), Seelhoff 2 (1884), Dickson 2 (1911), Mason 14 (1921), Escott 31 (1930). (Received March 25, 1930.)

272. Dr. W. D. Baten: *The probable error of certain functions of the errors.*

This paper presents with proof and applications several theorems pertaining to the probable error of certain functions of the errors made in measurements. It is assumed that a is the true value of the quantity to be measured, m the measurement or observation. If the measurements are multiplied by a constant b, the probable value of the functions of the error of bm will differ from the probable value of these same functions of the error of m. This paper shows that there are values of b such that the probable value of certain functions of the error of bm is less than the probable value of these functions of the error of m. The probable value of certain functions of the error of the mean is also treated. General frequency laws for the error are used which include the discrete, the continuous, and a combination of the discrete and the continuous cases. Stieltjes integrals are used to prove most of the theorems. Special cases are treated in the corollaries. (Received March 27, 1930.)

273. Mr. G. C. Munro: *Systems of partial differential equations with constant coefficients.*

This paper deals with a system of linear homogeneous partial differential equations in n dependent variables, with constant coefficients, namely, the system

$$\frac{\partial x_i}{\partial u}=\sum_{j=1}^{n}a_{ij}x_j, \qquad \frac{\partial x_i}{\partial v}=\sum_{j=1}^{n}b_{ij}x_j, \qquad (i=1, \cdots, n),$$

where the a_{ij} and b_{ij} are constants. All possible conditions under which common solutions for the system may exist are discussed, and in each case the solutions are obtained explicitly. It is shown that all common solutions are exponential in form, that is, they consist of exponentials whose coefficients are polynomials in the independent variables. It is further shown that there exist fundamental sets of common solutions for the system if and only if multiplication between the matrices of the coefficients in the two sets of equations is commutative. If these matrices, A and B, are not commutative, the number of common solutions for the system is at most equal to the number of dependent variables, n, minus the rank of the matrix $AB-BA$. (Received March 15, 1930.)

274. Dr. M. G. Boyce: *An envelope theorem and necessary conditions for a problem of Mayer with one variable end point.*

Let y denote the set $y_1, \cdots, y_n$. The problem studied is that of finding among the arcs $y=y(x)$ which satisfy a system of differential equations $\phi_\alpha(x, y, y')=0$, $\alpha=1, \cdots, m<n$, one which has a fixed initial point (x_1, y_1) and whose end point $[x_2, y(x_2)]$ minimizes the first of a set of functions $f_\sigma[x_2, y(x_2)]$, $\sigma=0, 1, \cdots, r<n$, while making the rest vanish. Multiplier

rules with transversality conditions have been derived for more general problems by Bolza and Bliss. In this paper further necessary conditions are given analogous to those of Weierstrass and Legendre. An envelope theory is then developed and is shown to lead to a Jacobi condition. Attention is given to the distinction between normal and abnormal admissible arcs, and it is found possible to make the proofs with weaker normality hypotheses than are customarily used in the Lagrange problems. (Received March 25, 1930.)

275. Professor L. E. Dickson: *Researches in the theory of numbers.*

New results on quadratic forms and diophantine equations in three or more variables, and on Waring's problem and its generalizations are given. The former appear in the author's current book, *Studies in the Theory of Numbers* (University of Chicago Press). (Received April 16, 1930.)

276. Mr. A. E. Ross: *On certain universal and zero indefinite ternary quadratic forms.*

In this paper we consider properly primitive indefinite ternary quadratic forms f with the invariants $\Omega=\pm 1$ and $\Delta=\mp D$, where $D>0$ and $\not\equiv 0 \pmod 4$. We show that such a form f is universal if and only if $(F/\rho)=(-\Omega/\rho)$ for every odd prime factor ρ of D, and that every universal form f is a zero form. Here F is the reciprocal of f. The conditions $\Omega=\pm 1$ and $\Delta\not\equiv 0 \pmod 4$ are necessary in order that the form f be universal. This last fact was communicated to the writer by Mr. A. Oppenheim since the paper was written. Therefore the above theorem is of interest as it furnishes an easily applicable criterion for universality of any properly primitive indefinite ternary quadratic form. Write $D=PQ^2$, where Q is the largest square divisor of D, $P=2^\alpha p_1\cdots p_\pi$, where $p_1, \cdots, p_\pi$ are positive odd primes. Let σ be the number of distinct odd primes dividing Q but not P. We show that a form f is a zero form if and only if $(F/p_j)=(-\Omega/p_j)$ $(j=1, \cdots, \pi)$, that there are 2^σ classes of zero forms f, and that the absolute value of the minimum of every form f which is not a zero form is unity. (Received April 18, 1930.)

277. Mr. A. E. Ross: *On universal indefinite quaternary quadratic forms* $\phi=f(x, y, z)+\alpha w^2$.

C. G. Latimer (Annals of Mathematics, (2), vol. 28 (1926–7), p. 327) had shown that the form $x^2+y^2-\alpha z^2-\alpha w^2$ represents all integers if $\alpha\leqq 163$ is of the form $4k+3$ and is the product of distinct positive odd primes. The same was shown by R. H. Marquis (Dissertation, University of Chicago, 1929) for $\alpha<81\equiv 1 \pmod 4$ and containing no square factors. Employing results of a former paper (abstract No. 36–1–38) we here prove the following general theorem which includes the results above as a very special sub-case. Let $f(x, y, z)$ be an indefinite ternary quadratic form whose determinant D is the product of distinct odd primes. Let F be the reciprocal of f. Let α be an integer $\not\equiv 0 \pmod 8$ and not divisible by the square of any prime factor p of D for which $(F/p)=-(-\Omega/p)$. Then the form $\phi=f(x, y, z)-\alpha w^2$ represents all integers. Here Ω is the first invariant of f. (Received April 18, 1930.)

278.* Professor E. T. Bell: *Residues of certain binomial coefficients for composite moduli.*

If $a, b, \cdots, c$ are integers greater than zero, and $p, q, \cdots, r$ are distinct positive primes, there is a simple explicit formula for the residue modulo $p^\alpha q^\beta \cdots r^\rho$ of the coefficient of x^h in the expansion of $(1+x)$ to the power $ap+bq+\cdots+cr$. The formula holds also when some but not all of $a, b, \cdots, c$ are zero. (Received May 13, 1930.)

279. Professor E. T. Bell: *Unique decomposition.*

A complete theory is developed around the following problem, of which the theory provides a denumerable infinity of distinct solutions. A set closed under a single operation, called composition, is called an *ovum* if with respect to the operation the elements of the set satisfy the associative and commutative laws. Suppose that in an ovum there is a denumerable infinity of elements, or a finite number of elements, such that any element in this set is uniquely composable from elements of a subset of the set, and no element in the subset is composable out of elements of the subset. This obviously is the situation of unique factorization into primes in its lowest terms. From the data alone no more arithmetic is possible since, if composition be identified with the commutative and associative properties of multiplication or of addition, it is automatically identified with those of both. A complete arithmetic is both additive and multiplicative. The problem is, from the data as above to define with respect to the elements and their law of unique composition new elements which shall have a complete arithmetic, order relations included, such that any relation in the complete theory has a unique interpretation in terms of the data. In giving the solutions it became necessary to devise a theory of arithmetic divisibility for divisors of zero. The theory contains many new (but simple) concepts. In particular it provides an effective definition for the addition of ideals which is given in detail as an illustration in the next paper. (Received May 13, 1930.)

280. Professor E. T. Bell: *Rings whose elements are ideals.*

Given only the unique decomposition of an integer, considered as a principal ideal, in any algebraic number field, it is required (1) to order the ideals of the field; (2) from (1) and the original data to define addition, multiplication and subtraction of ideals so that, under these operations, which have abstractly all the properties of the similarly named operations in a ring, the ideals of the field shall form a ring; (3) to interpret the solutions of (2) in terms of the original data. This problem is solved as an example of the preceding theory. Addition in the solution is multiplication in the original field; multiplicaton is a new process which has close affinities with the fundamental properties of the G.C.D. and L.C.M. as defined for Dedekind ideals. In this, as in the general theory, it was necessary to discuss arithmetical divisibility for divisors of zero. The solution gives automatically the properties of "greater" and

* The papers beyond this point are to be read at meetings of which the reports have not yet been published.

"less," abstractly identical with the like for real numbers, with reference to algebraic numbers and ideals. This does not depend upon the norm, but upon the rational integers occurring as coefficients in the basis representation of integers or ideals. This again is but an instance of the theory of inequality for matrices whose elements are real numbers. (Received May 13, 1930.)

281. Professor E. T. Bell: *A theory of certain fields of points.*

Another type of solution of the problem of unique decomposition is presented. This is geometrical in character and refers to fields (the rational operations being given suitable geometrical interpretations) of points lying on a one dimensional variety in a space of n dimensions. The points on a sphere in n-space are represented parametrically by a well known transformation due to Green (Cambridge Philosophical Transactions, 1835). The set of all points on the sphere whose parameters are the coordinates of points on a sphere in the next lower space is said to define a set of cospherical points on the original sphere. The process is continued until a circle is reached. The points of the circle are then considered as the elements in a field after being placed in one to one correspondence, by a certain transcendental transformation, with all the points on the real axis. The theory concerning the circle or the points in the closed interval from 0 to 1 was given in a previous paper, *The real unit segment as a number field* (Abstract No. 36–5–218). The process is now reversed and fields of points in n-space are generated. Algebraically, all the operations and theorems can be interpreted as relations between ordered sets of n real numbers. Order relations for such ordered sets treated as elements are essential to the theory; they are abstractly identical with "greater" and "less" as referred to real numbers. (Received May 13, 1930.)

282. Dr. Arthur B. Brown: *An extension of the Alexander duality theorem.*

Let S be an n-manifold with the Betti numbers of an n-sphere. Let D be a proper sub-set of the points of S, possessing finite or infinite topological Betti numbers; and let K be a closed point set on a part of D which is open with respect to S. Then the following relations hold, where $R_{n-i-1}(K)$ denotes the Vietoris Betti number, and the other terms denote topological Betti numbers:

$$R_i(D - K) = R_i(D) + R_{n-i-1}(K), \quad i = 0, 1, \cdots, n - 1.$$

This theorem is proved by use of the Alexander duality theorem (as extended by Alexandroff, Lefschetz and Alexander) and a theorem about overlapping complexes. Another proof has been found which is longer, but depends only on the Alexander theorem. (Received May 13, 1930.)

283. Professor Daniel Buchanan: *Periodic orbits in the problem of three bodies with repulsive and attractive forces.*

Two repellant particles under the attraction of a central nucleus will move in circular orbits the planes of which are parallel. The line joining the centres of these circles is normal to their planes and is bisected by the nucleus. The paper deals with periodic oscillations near these circular orbits. The configuration of the three bodies is similar to that of the helium atom. (Received May 13, 1930.)

284. Professor Daniel Buchanan: *Periodic orbits in the problem of four bodies with repulsive and attractive forces.*

The problem considered deals with the motion of two repellant particles which move subject to the attraction of two nuclei of equal mass. Periodic orbits for the particles are obtained which lie in the right bisector plane of the join of the nuclei. The configuration of the four bodies is similar to that of the normal hydrogen molecule. (Received May 13, 1930.)

285. Professor Florian Cajori: *The word "logarithm" used before the time of Napier.*

"Logarithm" is used in the compound word "logarithmanteia" by Casper Peucer in his *Commentarius* on methods of divination (Wittenberg, 1553). This fact I owe to Roscoe Lamont of Washington, D. C. Peucer associates the letters of the alphabet with triangular numbers. Divination is achieved by passing from numbers to letters or conversely. "Logarithmanteia" is derived from "logos" (word), "arithmos" (number) and "manteia" (divination). With Napier "logos" means ratio. The Englishman John Gaule in 1652 used "logarithmancy" in Peucer's sense of divination "by logarithmes." Peucer is mentioned by Robert Burton in 1621. It is possible that Napier got his word "logarithm" (1614) from Peucer. (Received May 13, 1930.)

286. Professor A. F. Carpenter: *Note on ruled surfaces and their developables.*

Each plane π fixed in position with respect to a ruled surface R determines with R two developable surfaces. One cuts π in the curve of intersection of π and R, the other cuts π in a curve whose points are those which correspond to π in the null-systems of the osculating linear complexes of R, and they intersect each other in a curve whose points are the poles of π with respect to the quadrics which osculate R along its line elements.

The proof makes use of conditions on the coordinates of π developed by the author in a previous paper (Annali di Matematica, (3), vol. 27 (1917), p. 285). (Received May 13, 1930.)

287. Ronald M. Foster: *Mutual impedances of grounded circuits.*

The mutual impedance between circuits consisting of insulated wires of negligible diameter lying on the surface of the earth and grounded at their terminals is

$$\begin{aligned} Z_{12} &= \frac{1}{2\pi\lambda}\int\int\left\{\frac{d^2}{dSds}\left(\frac{1}{r}\right)+\frac{\cos\epsilon}{r^2}[1-(1+\gamma r)e^{-\gamma r}]\right\}dSds \\ &= \frac{1}{2\pi\lambda}\left(\frac{1}{Aa}-\frac{1}{Ab}-\frac{1}{Ba}+\frac{1}{Bb}\right)+i\omega N_{Sa} \\ &\quad +(1-i)\frac{1}{3}(8\pi\lambda\omega^3)^{1/2}AB\cdot ab\cdot\cos\theta+\cdots \\ &= \int\left[\frac{1}{\pi\lambda X^2}-\frac{\gamma}{\pi\lambda X}K_1(\gamma x)\right]\cos\epsilon ds+\cdots. \end{aligned}$$

The earth is assumed flat, semi-infinite in extent, and of uniform conductivity λ. All displacement currents are neglected,—the frequency $\omega/(2\pi)$ thus being limited to relatively low values. The propagation constant is $\gamma = (i4\pi\lambda\omega)^{1/2}$. In the first form the integrations are extended over the two wires S and s, their elements dS and ds being separated by the distance r and making the angle ϵ with each other. The second form gives the first three terms in the expansion for low frequencies; N_{S_s} is the mutual Neumann integral between the two wires S and s, which extend from A to B and from a to b, respectively, and θ is the angle between the straight lines AB and ab. The third form gives the first term in the expansion for a long straight wire S and any wire s located near the mid-point of S, x and ϵ being respectively the distance from the element ds to S and the angle between ds and S. (Received May 13, 1930.)

288. Professor Raymond Garver: *An application of a theorem of Junker.*

Considerable use has been made by Hermite, Cayley, Brioschi and others of transformations on algebraic equations which lead to transformed equations whose coefficients have certain invariantive properties. However, one very general transformation of this type, due to Junker, has apparently escaped attention almost entirely. The present paper mentions several problems in which it may be of value, and considers in detail its use in the reduction of the general quartic to binomial form. The method employed seems to be of some interest apart from its connection with Junker's theorem; in fact, the main result of this paper can be obtained quite conveniently without reference to the latter. (Received May 13, 1930.)

289. E. R. Hedrick: *Varieties of polar singularities of non-analytic functions.*

In this paper the author shows the possibilities of different varieties of singularities of non-analytic functions which have some of the properties usually associated with poles of analytic functions. For one type, called analytic poles, the author shows that the classical theorems regarding functions which have only poles remain valid for non-analytic functions. (Received May 13, 1930.)

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Annali di Matematica: Ser. 1; Ser. 2, Vols. 1-4, 22, 24-26; Ser. 3, Vols. 14, 16.

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Meetings of the Society have been fixed at the following times and places:

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Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, Calif., not later than May 23. For publication in advance of the meeting, abstracts must be received on or before May 13.

PROVIDENCE, RHODE ISLAND, SUMMER MEETING AND COLLOQUIUM, September 9-12, 1930.

Abstracts must be in the hands of the Secretary of the Society, Professor R. G. D. Richardson, 501 West 116th Street, New York City, not later than August 11. In order to be printed in the Bulletin in advance of the meeting, abstracts must be in the hands of the Secretary not later than July 7. Professor Solomon Lefschetz will deliver a series of colloquium lectures on *Topology*. Professors T. H. Hildebrandt and J. D. Tamarkin will deliver addresses on topics in the general theory of linear operations and integral equations.

NEW YORK CITY, October 25, 1930.

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